Coursefile contents:

- 1. Cover Page
- 2. Syllabus copy
- 3. Vision of the department
- 4. Mission of the department
- 5. PEOs and POs
- 6. Course objectives and outcomes
- 7. Brief note on the importance of the course and how it fits in to the curriculum
- 8. Prerequisites
- 9. Instructional Learning Outcomes
- 10.Course mapping with PEOs and POs
- 11.Class Time Table
- 12.Individual Time Table
- 13.Lecture schedule with methodology being used/adopted
- 14.Detailed notes
- 15.Additional/missing topics
- 16.University previous Question papers
- 17.Question Bank
- 18.Assignment topics
- 19.Unit wise questions
- 20.Tutorial problems
- 21.Known gaps
- 22.Discussion topics
- 23.References, Journals, websites and E-links
- 24. Quality measurement Sheets
	- a. course and survey
	- b. Teaching evaluation
- 25. Student List
- 26. GroupWise Student List for discussion topics

2. Syllabus:

1. Vision of the department

Vision and Mission of the institute

The Mission of the institute

Our mission is to become a high quality premier educational institution, to create technocrats, by ensuring excellence, through enriched knowledge, creativity and self-development.

The Vision of the institute

Geethanjali visualizes dissemination of knowledge and skills to students, who would eventually contribute to the well being of the people of the nation and global community.

DEPARTMENT OF EEE

Department of Electronics and Electronics Engineering is established in the year 2006 to meet the requirements of the Electrical and Electronic industries such as Vijay electrical, BHEL, BEL and society after the consultation with various stakeholders.

Vision of EEE

To provide excellent Electrical and electronics education by building strong teaching and research environment

3.Mission of the department

Mission of EEE

- i) To offer high quality graduate program in Electrical and Electronics education and to prepare students for professional career or higher studies.
- ii) The department promotes excellence in teaching, research, collaborative activities and positive contributions to society

2. PEOs and Pos Program Educational Objectives

Program Educational Objectives of the UG Electrical and Electronics Engineering are:

PEO 1. Graduates will excel in professional career and/or higher education by acquiring knowledge in Mathematics, Science, Engineering principles and Computational skills.

PEO 2. Graduates will analyze real life problems, design Electrical systems appropriate to the requirement that are technically sound, economically feasible and socially acceptable.

PEO 3.Graduates will exhibit professionalism, ethical attitude, communication skills, team work in their profession, adapt to current trends by engaging in lifelong learning and participate in Research & Development.

Programme Outcomes

The Program Outcomes of UG in Electrical and Electronics Engineering are as follows:

PO 1. An ability to apply the knowledge of Mathematics, Science and Engineering in Electrical and Electronics Engineering.

PO 2. An ability to design and conduct experiments pertaining to Electrical and Electronics Engineering.

PO 3. An ability to function in multidisciplinary teams

PO 4. An ability to simulate and determine the parameters such as nominal voltage, current, power and associated attributes.

PO 5. An ability to identify, formulate and solve problems in the areas of Electrical and Electronics Engineering.

PO 6. An ability to use appropriate network theorems to solve electrical engineering problems.

PO 7. An ability to communicate effectively.

PO 8. An ability to visualize the impact of electrical engineering solutions in global, economic and societal context.

PO 9. Recognition of the need and an ability to engage in life-long learning.

PO 10 An ability to understand contemporary issues related to alternate energy sources.

PO 11 An ability to use the techniques, skills and modern engineering tools necessary for Electrical Engineering Practice.

PO 12 An ability to simulate and determine the parameters like voltage profile and current ratings of transmission lines in Power Systems.

PO 13 An ability to understand and determine the performance of electrical machines namely speed, torque, efficiency etc.

PO 14 An ability to apply electrical engineering and management principles to Power Projects

Mapping of Course with Programme Educational Objectives:

6.Course Objective & Course outcomes:

1. To equip the students with the knowledge and techniques of analyzing Three phase electrical circuits.

2. Students learn network function representation.

3. Students learn to characterize and analyse networks in both the time and complex frequency domain.

4. Students learn the concepts of Two-port Network parameters.

5. To introduce the concept of DC and AC transient analysis.

6. To introduce the student to different types of filters.

7. To learn about the use of mathematics, need of different transforms and usefulness of differential equations for analysis of networks.

8. With this the students will have the knowledge of how to evaluate and analyze any complex network.

Subject: NETWORK THEORY

CO 1: Learner will be able to apply knowledge of mathematics to solve numerical based on network simplification and it will be used to analyze the same.

CO 2: Analyze AC and DC transient response of resistance, inductance and capacitance in terms of impedance.

CO 3: Analyze Three phase circuits.

CO 4: Understand, and calculate the initial conditions of RL, RC circuits.

CO 5: To formulate, solve the differential equations for RL, RC, and RLC circuits and carry out the transient analysis.

CO 6: Understand, analyze and design prototype LC filters.

CO 7: Characterize and model the network in terms of all network parameters and analyze.

CO 8: Understand and formulate the network transfer function in s-domain and pole, zero concept.

Mapping of Course outcomes with Programme outcomes:

*When the course outcome weightage is < 40%, it will be given as moderately correlated (1). *When the course outcome weightage is $>40\%$, it will be given as strongly correlated (2).

7.Instructional Learning Outcomes:

8. Class Time Table:

9. Individual Time Table

Geethanjali college of Engineering & Technology

Department of Electrical and Electronics Engineering

Year& Semester to whom subject is Offered______ II Year – II sem_

Name of the Subject Network Theory

Name of the Faculty: Dr. S. Radhika **Designation** Professor

Department:
Electrical and Electronics Engineering

Introduction to the Subject

This subject give the brief description the property of the circuit element and their behavior for various excitations. It also throw a glance on the on the Design and operation of the two port networks and different types of filters

Objectives of the subject:

This course introduces the basis concepts of circuit analysis which is the foundation for all subjects of the Electrical Engineering discipline. The emphasis of this course if laid on the basic analysis of circuits which includes three phase circuits, transient analysis for both DC and Sinusoidal excitation

This course also introduces the concept of network functions, complex frequency, transform impedance and transform circuits. It also gives a brief description of Fourier analysis of AC circuits.

JNTU Syllabus with Additional Topics:

Unit wise Summary

Micro plan

Detailed Lecture notes containing

1.PPTS

2.OHP slides

- 3.Subjective type questions (approximately 5 to 8 /unit)
- 4.Objective type questions (approximately 20 to 30 /unit
- 5.Any simulations

Course Review (By the concerned Faculty):

- (i) Aims
- (ii) Sample Check
- (iii)End of the course report by the concerned faculty

GUIDELINES:

Distribution of periods:

Detailed notes:

Unit-1

Detailed Notes:

Unit-01

BALANCED THREE-PHASE AC CIRCUIT

- **Balanced Three-Phase Voltage Sources** \bullet **Delta Connection Star Connection**
- **Balanced 3-phase Load** \bullet **Delta Connection Star Connection**
- **Power in a Balanced Phase Circuit** \bullet

Balanced Three Phase Voltages Three-phase voltage sources

a) wye-connected source

b) delta-connected source

If the voltage source have the same amplitude and frequency ω and are out of phase with each other by 120°, the voltage are said to be balanced.

$$
|V_{an}| = |V_{bn}| = |V_{cn}|
$$
 $V_{an} + V_{bn} + V_{cn} = 0$

Balanced phase voltages are equal in magnitude and out of phase with each other by 120°

Balanced Three Phase Loads

Two possible three-phase load configurations:

a) a Star or Y-connected load

b) a delta-connected load

For a balanced wye connected load:

$$
Z_1 = Z_2 = Z_3 = Z_Y
$$

$$
Z_Y = \frac{1}{3}Z_{\Delta}
$$

$$
Z_{a} = Z_{b} = Z_{c} = Z_{A}
$$

$$
Z_{A} = 3Z_{Y}
$$

For a balanced delta connected load:

Example 1

Determine the phase sequence of the set of voltages

 $v_{an} = \sqrt{2} 200 \cos(\omega t + 10^{\circ})$

 $v_{bn} = \sqrt{2} 200 \cos(\omega t - 230)$, $v_{cn} = \sqrt{2} 200 \cos(\omega t - 110)$

Solution:

The voltages can be expressed in phasor form as

We notice that V_{an} leads V_{cn} by 120° and V_{cn} in turn leads V_{bn} by 120^o. $V_{an} = 200\angle 10^{\circ}V$ $V_{bn} = 200\angle -230^{\circ}V$ $V_{cn} = 200\angle -110^{\circ}V$ Hence, we have an *acb* sequence.

Given that $V_{bn} = 110\angle 30^{\circ}V$, find Van and Vcn, assuming a positive (abc) sequence.

Answer:

$$
V_{\text{cm}} = 110\angle 150^{\circ}V \qquad V_{\text{cm}} = 110\angle -90^{\circ}V
$$

Balanced Y-Y Connection

A balanced Y-Y system is a three phase system with a balanced Y connected source and balanced Y connected load.

Balanced Wye-Wye Connection

- Z_s = Source impedance
- Z_i = Line impedance
- Z_{L} = Load impedance
- Z_v = Total impedance per phase
- $Z_v = Z_s + Z_t + Z_v$

$$
Z_{\rm Y} = Z_{\rm L}
$$

Balanced Y-Y Connection

Y-Y configuration Example:1

A balanced positive-sequence Y-connected 60 Hz three-phase source has phase voltage Va=1000V. Each phase of the load consists of a 0.1-H inductance in series with a $50-\Omega$ resistance.

Find the line currents, the line voltages, the power and the reactive power delivered to the load. Draw a phasor diagram showing line voltages, phase voltages and the line currents. Assuming that the phase angle of V_{an} is zero.

2- A Y-connected balanced three-phase generator with an impedance of $0.4+j0.3 \Omega$ per phase is connected to a Y-connected balanced load with an impedance of $24 + i19 \Omega$ per phase. The line joining the generator and the load has an impedance of $0.6 + j0.7$ Q per phase.

Assuming a positive sequence for the source voltages and that

 $V_{an} = 120 \angle 30^{0} V$

Find: (a) the line voltages (b) the line currents

Balanced Y-Delta Connection

A balanced $Y - \Delta$ system consists of balanced Y connected source feeding a balanced Δ connected load.

 $I_{CA} = I_{AB} Z - 240$
Researced Y-Delta Connect $I_a = I_{AB} - I_{CA} = I_{AB} (1 - 1 \angle - 240^\circ)$ I_{CA} 30 $I_a = I_{AB} \sqrt{3} \angle -30^0$ I_{AB} Magnitude line currents: $\boxed{I_L = I_p \sqrt{3}}$
 $I_L = |I_a| = |I_b| = |I_c|$ $I_p = |I_{AB}| = |I_{BC}| = |I_{CA}|$ A single phase equivalent circuit

Y-Delta configuration: Example 3

1- A balanced abc sequence Y-connected source with $V_{an} = 100 \angle 10^0 V$

is connected to a Δ -connected balanced load (8+j4) Ω per phase. Calculate the phase and line currents.

2-One line voltage of a balanced Y-connected source is $V_{AB} = 180\angle -20^{\circ}V$

If the source is connected to a Δ -connected load of $20 \angle 40^{\circ} \Omega$, find the phase and line currents.

Assume the abc sequence.

Balanced Delta-Delta Connection

A balanced Δ - Δ system is the one in which both balanced source and balanced load are Δ connected.

1- A balanced Δ connected load having an impedance of 20-j15 Ω is connected to a Δ connected, positive sequence generator having

$$
V_{ab} = 330 \angle 0^0 V
$$

Calculate the phase currents of the load and the line currents.

2- A positive-sequence, balanced -connected source supplies a balanced Δ -connected load. If the impedance per phase of the load is $18+j12 \Omega$ and $I_a = 22.5 \angle 35^\circ A$, find I_{AB} and V_{AB} .

Y-Delta configuration: Example 5

1-A balanced Y connected load with a phase resistance of 40 Ω and a reactance of 25 Ω is supplies by a balanced, positive sequence Δ connected source with a line voltage of 210 V. Calculate the phase currents. Use V_{ab} as reference.

2-In a balanced -Y circuit, $V_{ab} = 240\angle 15^{\circ}V$ and $\mathbf{Z}_r = (12 + j15) \Omega$. Calculate the line currents.

POWER IN A BALANCED SYSTEM

For Y connected load, the phase voltage: $v_{AN} = \sqrt{2}V_p \cos \omega t$, $v_{BN} = \sqrt{2}V_p \cos(\omega t - 120^\circ)$, $v_{CN} = \sqrt{2}V_p \cos(\omega t + 120^\circ)$ If $Z_y = Z \angle \theta$ Phase current lag phase voltage by θ . $i_a = \sqrt{2}I_p \cos(\omega t - \theta)$ The phase current: $i_b = \sqrt{2}I_p \cos(\omega t - \theta - 120^\circ)$

 $i_e = \sqrt{2}I_p \cos(\omega t - \theta + 120^\circ)$

POWER IN A BALANCED SYSTEM

Total instantaneous power:

$$
p = p_a + p_b + p_c = v_{AN}i_a + v_{BN}i_b + v_{CN}i_c
$$

$$
p = 3V_pI_p \cos \theta
$$

Average power per phase:

Reactive power per phase:

$$
\mathbf{P}_{\mathrm{p}} = \mathbf{V}_{\mathrm{p}} \mathbf{I}_{\mathrm{p}} \cos \theta
$$

Apparent power per phase:

$$
\mathbf{S}_{\mathbf{p}} = \mathbf{V}_{\mathbf{p}} \mathbf{I}_{\mathbf{p}}
$$

 $Q_p = V_p I_p \sin \theta$

Complex power per phase:

$$
S_{p} = P_{p} + jQ_{p} = V_{p}I_{p}^*
$$

POWER IN A BALANCED SYSTEM

Total average power:

$$
P = 3Pp = 3VpIp \cos \theta = \sqrt{3}VLIL \cos \theta
$$

Total reactive power:

$$
Q = 3Q_p = 3V_pI_p \sin \theta = \sqrt{3}V_LI_L \sin \theta
$$

Total complex power:

$$
S=3S_{_p}=3V_{_p}I_{_p}^*=3I_{_p}^2Z_{_p}=\frac{3V_{_p}^2}{Z_{_p}^*}
$$

$$
S = P + jQ = \sqrt{3}V_{L}I_{L}\angle\theta
$$

Power: Example 6

1-A three-phase motor can be regarded as a balanced Y-load. A three-phase motor draws 5.6 kW when the line voltage is 220 V and the line current is 18.2 A. Determine the power factor of the motor.

2- Calculate the line current required for a 30-kW three-phase motor having a power factor of 0.85 lagging if it is connected to a balanced source with a line voltage of 440 V.

UNIT-02

DC AND AC TRANSIENT ANALYSIS

Introduction

When a d.c. voltage is applied to a capacitor C and resistor R connected in series, there is a short period of time immediately after the voltage is connected, during which the current flowing in the circuit and voltages across C and R are changing.

Similarly, when a d.c. voltage is connected to a circuit having inductance L connected in series with resistance R, there is a short period of time immediately after the voltage is connected, during which the current flowing in the circuit and the voltages across L and R are changing.

These changing values are called transients.

Charging a capacitor

(a) The circuit diagram for a series connected $C-R$ circuit is shown in Figure 17.1. When switch S is closed then by Kirchhoff's voltage law:

 $V = v_C + v_R$

(b) The battery voltage V is constant. The capacitor voltage v_C is given by q/C , where q is the charge on the capacitor. The voltage drop across R is given by iR , where i is the current flowing in the circuit. Hence at all times:

$$
V = \frac{q}{C} + iR
$$

At the instant of closing S, (initial circuit condition). assuming there is no initial charge on the capacitor, q_0 is zero, hence v_{Co} is zero. Thus from equation (17.1) , $V = 0 + v_{\text{Ro}}$, i.e. $v_{\text{Ro}} = V$. This shows that the resistance to current is solely due to R , and the initial current flowing, $i_0 = I = V/R$.

- (c) A short time later at time t_1 seconds after closing S. the capacitor is partly charged to, say, q_1 coulombs because current has been flowing. The voltage v_{C1} is now q_1/C volts. If the current flowing is i_1 amperes, then the voltage drop across R has fallen to i_1R volts. Thus, equation (17.2) is now $V = (q_1/C) + i_1 R$.
- (d) A short time later still, say at time t_2 seconds after closing the switch, the charge has increased to q_2 coulombs and v_C has increased to q_2/C volts. Since $V = v_C + v_R$ and V is a constant, then v_R decreases to i_2R . Thus v_C is increasing and i and v_R are decreasing as time increases.
- (e) Ultimately, a few seconds after closing S, (i.e. at the final or steady state condition), the capacitor is fully charged to, say, Q coulombs, current no longer flows, i.e. $i=0$, and hence $v_R = iR = 0$. It follows from equation (17.1) that $v_C = V$.

(f) Curves showing the changes in vc , v_R and i with time are shown in Figure 17.2.

The curve showing the variation of v_C with time is called an exponential growth curve and the graph is called the 'capacitor voltage/time' characteristic. The curves showing the variation of v_R and i with time are called exponential decay curves, and the graphs are called 'resistor voltage/time' and 'current/time' characteristics respectively. (The name 'exponential'

time t₁ seconds. Let the voltage be varied so that the current flowing in the circuit is constant.

- (c) Since the current flowing is a constant, the curve will follow a tangent, AB, drawn to the curve at point A.
- (d) Let the capacitor voltage v_C reach its final value of V at time t_2 seconds.
- (e) The time corresponding to $(t_2 t_1)$ seconds is called the time constant of the circuit, denoted by the Greek letter 'tau', τ . The value of the time constant is CR seconds, i.e. for a series connected $C-R$ circuit,

time constant $\tau = CR$ seconds

Since the variable voltage mentioned in para (b) above can be applied to any instant during the transient change, it may be applied at $t = 0$, i.e. at the instant of connecting the circuit to the supply. If this is done, then the time constant of the circuit may be defined as:

'the time taken for a transient to reach its final state if the initial rate of change is maintained'.

Transient curves for a C-R circuit

There are two main methods of drawing transient curves graphically, these being:

A circuit consists of a resistor connected in series with a 0.5μ F capacitor and has a time constant of 12 ms. Determine (a) the value of the resistor, and (b) the capacitor voltage 7 ms after connecting the circuit to a 10V supply.

(a) The time constant
$$
\tau = CR
$$
, hence $R = \frac{\tau}{C}$

i.e.
$$
R = \frac{12 \times 10^{-3}}{0.5 \times 10^{-6}} = 24 \times 10^{3} = 24 \text{ k}\Omega
$$

(b) The equation for the growth of capacitor voltage is:

$$
vc = V(1 - e^{-r/\tau})
$$

Since $\tau = 12 \text{ ms} = 12 \times 10^{-3} \text{ s}, V = 10 \text{ V}$ and
 $r = 7 \text{ ms} = 7 \times 10^{-3} \text{ s},$
then $v_C = 10 \left[1 - e^{-\frac{7 \times 10^{-3}}{12 \times 10^{-3}}} \right] = 10(1 - e^{-0.583})$
= 10(1 - 0.558) = 4.42 V

Alternatively, the value of v_C when t is 7 ms may be determined using the growth characteristic as shown in Problem 1.

A circuit consists of a 10μ F capacitor connected in series with a $25 \text{ k}\Omega$ resistor with a switchable 100V d.c. supply. When the supply is connected, calculate (a) the time constant, (b) the maximum current, (c) the voltage across the capacitor after $0.5 s$, (d) the current flowing after one time constant, (e) the voltage across the resistor after 0.1 s, (f) the time for the capacitor voltage to reach 45 V, and (g) the initial rate of voltage rise.

- (a) Time constant, $\tau = C \times R = 10 \times 10^{-6} \times 25 \times 10^3$ $= 0.25 s$
- (b) Current is a maximum when the circuit is first connected and is only limited by the value of resistance in the circuit, i.e.

$$
I_m = \frac{V}{R} = \frac{100}{25 \times 10^3} = 4 \,\text{mA}
$$

(c) Capacitor voltage, $v_C = V_m(1 - e^{\frac{-t}{k}})$

When time,
$$
t = 0.5
$$
 s, then
\n $v_C = 100(1 - e^{-\frac{0.5}{0.23}}) = 100(0.8647) = 86.47$ V

(d) Current, $i = I_m e^{-\frac{t}{\tau}}$

and when $t = \tau$, current, $i = 4e^{-\frac{\tau}{\tau}} = 4e^{-1} = 1.472$ mA

Alternatively, after one time constant the capacitor voltage will have risen to 63.2% of the supply voltage and the current will have fallen to 63.2% of its final value, i.e. 36.8% of I_m .

Hence, $i = 36.8\%$ of $4 = 0.368 \times 4 = 1.472$ mA

(e) The voltage across the resistor, $v_R = Ve^{-\frac{1}{2}}$ When $t = 0.1$ s, resistor voltage,

$$
v_R = 100e^{-\frac{0.1}{0.25}} = 67.03\,\mathrm{V}
$$

(f) Capacitor voltage, $v_C = V_m(1 - e^{-\frac{1}{x}})$

When the capacitor voltage reaches 45V, then: $45 = 100 \left(1 - e^{-\frac{t}{0.25}}\right)$ from which, $\frac{45}{100} = 1 - e^{-\frac{t}{0.25}}$ and $e^{-\frac{t}{0.25}} = 1 - \frac{45}{100} = 0.55$ Hence. $-\frac{t}{0.25}$ = ln 0.55 and time, $t = -0.25$ ln 0.55 $= 0.149 s$ (g) Initial rate of voltage rise $=\frac{V}{\tau_{av}} = \frac{100}{0.25} = 400 \text{ V/s}$ (i.e. gradient of the tangent at $t = 0$)

Time constant for an $L-R$ circuit

With reference to Section 17.3, the time constant of a series connected $L-R$ circuit is defined in the same way as the time constant for a series connected $C-R$ circuit, i.e. it is the time taken to reach its final value if the initial rate of change is maintained. Its value is given by:

time constant,
$$
\tau = L/R
$$
 seconds

Transient curves for an $L-R$ circuit

Transient curves representing the induced voltage/time, resistor voltage/time and current/time characteristics may be drawn graphically, as outlined in Section 17.4. A method of construction is shown in Problem 8. Each of the transient curves shown in Figure 17.11 have mathematical equations, and these are:

decay of induced voltage, $v_L = Ve^{(-Rt/L)} = Ve^{(-t/\tau)}$

growth of resistor voltage, $v_R = V(1 - e^{-Rt/L})$ $= V(1 - e^{-t/\tau})$ $t = I(1 - e^{-Rt/L})$ growth of current flow, $= I(1 - e^{-t/\tau})$

These equations are derived analytically in Chapter 45. The application of these equations is shown in Problem 10.

A relay has an inductance of 100 mH and
a resistance of 20 Ω . It is connected to a 60 V, d.c. supply. Use the 'initial slope and three point' method to draw the current/time characteristic and hence determine the value of current flowing at a time equal to two time constants and the time for the current to grow to $1.5 A.$

Be fore the current/time characteristic can be drawn, the time constant and steady state value of the current have to be calculated.

Time constant, $\tau = \frac{L}{R} = \frac{100 \times 10^{-3}}{20} = 5$ ms Final value of current, $I = \frac{V}{R} = \frac{60}{20} = 3$ A

The method used to construct the characteristic is the same as that used in Problem 2.

- (a) The scales should span at least five time constants
(horizontally), i.e. 25 ms , and 3 A (vertically).
- (b) With reference to Figure 17.12, the initial slope is obtained by making AB equal to 1 time constant, (5 ms) , and joining 0B.

At a time of 1 time constant, CD is $0.632 \times I =$ $0.632 \times 3 = 1.896$ A At a time of 2.5 time constants, EF is $0.918 \times I = 0.918 \times 3 = 2.754$ At a time of 5 time constants, GH = 3 A
(d) A smooth curve is drawn through points 0 , D , F and H and this curve is the current/time characteristic.

From the characteristic, when $t = 2\tau$, $i \approx 2.6$ A. [This may be checked by calculation using $i = I(1 - e^{-t/\tau})$, where $I = 3$ and $t = 2\tau$, giving $i = 2.59$ A]. Also, when the current is 1.5 A, the corresponding time is about 3.6 ms.

[This may be checked by calculation, using $i = I(1 - e^{-t/\tau})$] where $i = 1.5$, $I = 3$ and $\tau = 5$ ms, giving $t = 3.466$ ms.]

A coil of inductance 0.04 H and resistance $10 \, \Omega$ is connected to a 120 V, d.c. supply. Determine (a) the final value of current, (b) the time constant. of the circuit, (c) the value of current after a time equal to the time constant from the instant the supply voltage is connected, (d) the expected time for the current to
rise to within 1% of its final value.

(a) Final steady current,
$$
I = \frac{V}{R} = \frac{120}{10} = 12 \text{ A}
$$

\n(b) Time constant of the circuit, $\tau = \frac{L}{R} = \frac{0.04}{10}$

 $= 0.004$ s or 4 ms

- (c) In the time τ s the current rises to 63.2% of its final value of 12A, i.e. in 4 ms the current rises to 0.632 \times 12 = 7.58A
- (d) The expected time for the current to rise to within 1% of its final value is given by 5 s , i.e. $5 \times 4 = 20 \text{ ms}$.

Problem 10. The winding of an electromagnet has an inductance of $3H$ and a resistance of 15Ω . When it is Problem 10. connected to a 120 V, d.c. supply, calculate:

- (a) the steady state value of current flowing in the winding,
- (b) the time constant of the circuit,
- (c) the value of the induced e.m.f. after 0.1 s,
(d) the value of the induced e.m.f. after 0.1 s,
- value, and
- (c) the value of the current after $0.3 s$
- (a) The steady state value of current is $I = V/R$, i.e.
 $I = 120V15 = SA$
- (b) The time constant of the circuit, $\tau = L/R = 3/15 = 0.2$ s. Parts (c), (d) and (e) of this problem may be deter-
mined by drawing the transients graphically, as shown
- in Problem 7 or by calculation as shown below.
(c) The induced e.m.f., v_L is given by $v_L = Ve^{-t/\tau}$. The d.c. voltage V is 120 V, r is 0.1 s and r is 0.2 s, hence \mathbb{Z}/\mathbb{Z} . The

 $v_L = 120e^{-0.1/0.2} = 120e^{-0.5} = 120 \times 0.6065$ i.e. $v_L = 72.78$ V

(d) When the current is 85% of its final value, $i = 0.85I$.

Also,
$$
i = I(1 - e^{-t/\tau})
$$
, thus $0.85I = I(1 - e^{-t/\tau})$
\n $0.85 = 1 - e^{-t/\tau}$ and since $\tau = 0.2$,
\n $0.85 = 1 - e^{-t/0.2}$
\n $e^{-t/0.2} = 1 - 0.85 = 0.15$
\n $e^{t/0.2} = \frac{1}{0.15} = 6.6$

Taking natural logarithms of each side of this equation gives:

$$
\ln e^{t/0.2} = \ln 6.6
$$
, and by the laws of logarithms

$$
\frac{t}{0.2} \ln e = \ln 6.6
$$
. But $\ln e = 1$, hence

$$
t = 0.2 \ln 6.6
$$
 i.e. $t = 0.379$ s

(e) The current at any instant is given by $I = I(1 - e^{-t/\tau})$

When $I = 8$, $t = 0.3$ and $\tau = 0.2$, then $i = 8(1 - e^{-0.3/(0.2)}) = 8(1 - e^{-1.5})$ $= 8(1 - 0.2231) = 8 \times 0.7769$ i.e. $l = 6.215A$.

$UNIT-03$

TWO-PORT CIRCUITS

INTODUCTION

- . A pair of terminals through which a current may enter or leave a network is known as a *port*.
- · Two-terminal devices or elements (such as resistors, capacitors, and inductors) result in one-port networks.
- . Most of the circuits we have dealt with so far are two-terminal or one-port circuits, represented in the following figure $1(a)$:

INTRODUCTION

· The four-terminal or two-port circuits are involving op amps, transistors, and transformers, as shown in the following figure 1(b):

- · A port is an access to the network and consists of a pair of terminals.
- The current entering one terminal leaves through the other terminal so that the net current entering the port equals zero.

INTRODUCTION

- · A two-port network is an electrical network with two separate ports for input and output.
- A two-port network has *two terminal pairs* acting as access points.
- · Such networks are useful in communications, control systems, power systems, and electronics. For example, they are used in electronics to model transistors and to apply the analysis of transistor circuits.
- The current entering one terminal of a pair leaves the other terminal in the pair.

To characterize a two-port network requires that we relate the terminal quantities V1, V2, I1, and I2 in Fig. 1(b).

- . The various terms that relate these voltages and currents are called *parameters*.
- Our goal is to derive six sets of these parameters.
- . We will show the relationship between these parameters and how two-port networks can be connected in series, parallel, or cascade.

IMPEDANCE PARAMETERS

- Impedance and admittance parameters are commonly used in the synthesis of filters.
- A two-port network may be voltage-driven as in Fig. 2(a) or currentdriven as in Fig. 2(b).

The terminal voltages can be related to the terminal currents as:

$$
\begin{array}{c} \mathbf{V}_1 = \mathbf{z}_{11}\mathbf{I}_1 + \mathbf{z}_{12}\mathbf{I}_2 \\ \mathbf{V}_2 = \mathbf{z}_{21}\mathbf{I}_1 + \mathbf{z}_{22}\mathbf{I}_2 \end{array}
$$

or in matrix form as

$$
\begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{z}_{11} & \mathbf{z}_{12} \\ \mathbf{z}_{21} & \mathbf{z}_{22} \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix} = [\mathbf{z}] \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix}
$$

the Z terms are called the z parameters, and have units of ohms.

- The values of the parameters can be evaluated by setting $I1 = 0$
- or $I2 = 0$. Thus, 1

$$
\left\| \begin{array}{ll} {\bf z}_{11}=\left.\displaystyle \frac{{\bf V}_1}{{\bf I}_1}\right|_{{\bf I}_2=0},\qquad {\bf z}_{12}=\left.\displaystyle \frac{{\bf V}_1}{{\bf I}_2}\right|_{{\bf I}_1=0}\\ {\bf z}_{21}=\left.\displaystyle \frac{{\bf V}_2}{{\bf I}_1}\right|_{{\bf I}_2=0},\qquad {\bf z}_{22}=\left.\displaystyle \frac{{\bf V}_2}{{\bf I}_2}\right|_{{\bf I}_1=0} \end{array}\right.
$$

- Since the Z parameters are obtained by open-circuiting the input or output
- port, they are also called the open-circuit impedance parameters.
- \mathbf{z}_{11} = Open-circuit input impedance
- Z₁₂ = Open-circuit transfer impedance from port 1 to port 2
- \mathbf{z}_{21} = Open-circuit transfer impedance from port 2 to port 1
- Z₂₂ = Open-circuit output impedance

We obtain Z_{11} and Z_{21} by connecting a voltage V_1 (or a current source I₁) to port 1 with port 2 open-circuited as in Fig. 3(a) and finding I₁ and V₂

Similarly, we obtain \mathbf{z}_{12} and \mathbf{z}_{22} by connecting a voltage V_2 (or a current source I2) to port 2 with port 1 open-circuited as in Fig. 3(b) and finding Iz and V1

Sometimes Z₁₁ and Z₂₂ are called *driving-point impedances*, while Z₂₁ and Z₁₂ are called *transfer impedances*.

- When \mathbf{z}_{11} = \mathbf{z}_{22} , the two-port network is said to be *symmetrical*. ٠
- When the two-port network has no dependent sources, the transfer impedances \cdot are equal (Z12 = Z21), and the two-port is said to be reciprocal.
- A two-port is reciprocal if interchanging an ideal voltage source at one ٠ port with an ideal ammeter at the other port gives the same ammeter reading.
- The reciprocal network yields $V = 212I$ when connected as in Fig. 4(a), but yields $V = \textbf{Z}$ 21 When connected as in Fig. 4(b).

For a reciprocal network, the T-equivalent circuit in Fig.5(a) can be used.

· If the network is not reciprocal, a more general equivalent
network is shown in Fig. 5(b)

Example. 1

Determine the *z* parameters for the circuit in the following \bullet figure: 30 Ω 20Ω

CONTRACTOR

 $\overline{}$

Solution:

To determine z_{11} and $\mathsf{z}_{21},$ we apply a voltage source \mathbf{V}_1 to the input port and leave the output port open as in Fig. (a). Then,

$$
z_{11}=\frac{V_1}{I_1}=\frac{(20+40)I_1}{I_1}=60\ \Omega
$$

that is, z₁₁ is the input impedance at port 1.

$$
z_{21}=\frac{{\bf V}_2}{{\bf I}_1}=\frac{40{\bf I}_1}{{\bf I}_1}=40\ \Omega
$$

To find z_{12} and z_{22} , we apply a voltage source V_2 to the output port and leave the input port open as in Fig (b). Then,

$$
z_{12} = \frac{V_1}{I_2} = \frac{40I_2}{I_2} = 40 \ \Omega, \qquad z_{22} = \frac{V_2}{I_2} = \frac{(30 + 40)I_2}{I_2} = 70 \ \Omega
$$
Thus,

$$
[z]=\begin{bmatrix} 60 \ \Omega & 40 \ \Omega \\ 40 \ \Omega & 70 \ \Omega \end{bmatrix}
$$

Method 2

Alternatively, since there is no dependent source in the given circuit, $z_{12} = z_{21}$ and we can use Fig. 5(a). Comparing with Fig. $5(a)$, we get

Example 2

• Find I1 and I2 in the circuit of the following figure:

Example 2 Find I1 and I2 in the circuit in the following figure. Solution:

 \overline{a}

we can use Eq. (1) directly. Substituting the given Z parameters into Eq. (1),

$$
V_1 = 40I_1 + j20I_2
$$

$$
V_2 = j30I_1 + 50I_2
$$

since we are looking for I_1 and I_2 , we substitute

$$
V_1 = 100 \angle 0^{\circ}, \qquad V_2 = -10I_2
$$

into the above Eqs., which become

$$
100 = 40I1 + j20I2
$$

$$
- 10I2 = j30I1 + 50I2 \implies I1 = j2I2
$$

Substituting we get

$$
100 = j80I_2 + j20I_2 \implies I_2 = \frac{100}{j100} = -j
$$

\n
$$
I_1 = j2(-j) = 2.
$$
 Thus,
\n
$$
I_1 = 2/0^\circ A, \qquad I_2 = 1/ -90^\circ A
$$

 $\overline{}$

ADMITTANCE PARAMETERS

- \cdot Impedance parameters may not exist for a two-port network. So there is a need for an alternative means of describing such a network. This need is met by the second set of parameters.
- In either Fig. 6(a) or (b), the terminal currents can be expressed in $\ddot{}$ terms of the terminal voltages:
- The y terms are known as the *admittance parameters*
- (or, simply, y parameters) .
- and have units of siemens. ٠

 \cdot y_{11} = Short-circuit input admittance Y₁₂ = Short-circuit transfer admitte

from port 2 to port 1

· Y21 = Short-circuit transfer admitte from port 1 to port 2

- Yzz = Short-circuit output admittanc. \bullet
- We obtain y11 and y21 by connecting a current I1 to port 1 and short-circuiting port 2 as in Fig. $6(a)$, finding V₁ and I₂

When a two-port network has no dependent sources, the transfer admittances are equal (y12 = y21).

- . A reciprocal network (y12 = y21) can be modeled by the -equivalent circuit in Fig. 7(a).
- If the network is not reciprocal, a more general equivalent network is \blacksquare shown in Fig. 7(b).

Example .3

• Obtain the y parameters for the network shown in the following figure:

 (b)

Solution:

method1 ٠

To find y_{11} and y_{21} , short-circuit the output port and connect a current source I₁ to the input port as in Fig.(a) Since the 8- Ω resistor is short-circuited, the 2- Ω resistor is in parallel with the 4- Ω resistor. Hence,

$$
V_1 = I_1(4 \parallel 2) = \frac{4}{3}I_1, \qquad y_{11} = \frac{I_1}{V_1} = \frac{I_1}{\frac{4}{3}I_1} = 0.75 \text{ S}
$$

 $\overline{}$

By current division,

$$
-I_2 = \frac{4}{4+2}I_1 = \frac{2}{3}I_1, \qquad y_{21} = \frac{I_2}{V_1} = \frac{-\frac{2}{3}I_1}{\frac{4}{3}I_1} = -0.5 S
$$

Con.

Method 1 ٠

To get y_{12} and y_{22} , short-circuit the input port and connect a current source \mathbf{I}_2 to the output port as in Fig. (b). The 4- Ω resistor is short-circuited so that the 2- Ω and 8- Ω resistors are in parallel.

$$
V_2 = I_2(8 \parallel 2) = \frac{8}{5}I_2, \qquad y_{22} = \frac{I_2}{V_2} = \frac{I_2}{\frac{8}{5}I_2} = \frac{5}{8} = 0.625 S
$$

By current division,

$$
-I_1 = \frac{8}{8+2}I_2 = \frac{4}{5}I_2, \t y_{12} = \frac{I_1}{V_2} = \frac{-\frac{4}{5}I_2}{\frac{8}{5}I_2} = -0.5 S
$$

$$
V_1 = 0 \begin{array}{c} \frac{I_1}{\sqrt{6}} & \frac{2\Omega}{\sqrt{6}} \\ V_1 = 0 & \frac{2}{5}4\Omega & 8\Omega \frac{1}{\sqrt{6}} \\ V_2 = 0 & 0 \end{array}
$$

Method 2

 \bullet

· Alternatively, comparing the original figure with Fig. (a),

HYBRID PARAMETERS

This third set of parameters is based on making V1 and Iz the \bullet dependent variables.

$$
\begin{array}{c} \bf{V}_1 = \bf{h}_{11} \bf{I}_1 + \bf{h}_{12} \bf{V}_2 \\ \bf{I}_2 = \bf{h}_{21} \bf{I}_1 + \bf{h}_{22} \bf{V}_2 \end{array}
$$

or in matrix form.

$$
\begin{bmatrix} \mathbf{V}_1 \\ \mathbf{I}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{h}_{11} & \mathbf{h}_{12} \\ \mathbf{h}_{21} & \mathbf{h}_{22} \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{V}_2 \end{bmatrix} = [\mathbf{h}] \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{V}_2 \end{bmatrix}
$$

- · The *h* terms are known as the *hybrid parameters* (or, *h parameters*)
- The ideal transformer can be described by the hybrid parameters. \bullet .

The values of the parameters are determined as

$$
\begin{aligned} \mathbf{h}_{11} &= \left. \frac{\mathbf{v}_1}{\mathbf{I}_1} \right|_{V_2=0}, \qquad \mathbf{h}_{12} = \left. \frac{\mathbf{v}_1}{\mathbf{v}_2} \right|_{\mathbf{I}_1=0} \\ \mathbf{h}_{21} &= \left. \frac{\mathbf{I}_2}{\mathbf{I}_1} \right|_{V_2=0}, \qquad \mathbf{h}_{22} = \left. \frac{\mathbf{I}_2}{\mathbf{v}_2} \right|_{\mathbf{I}_1=0} \end{aligned}
$$

- \cdot h₁₁ = Short-circuit input impedance
- . hiz = Open-circuit reverse voltage gain
- \cdot h₂₁ = Short-circuit forward current gain
- hzz = Open-circuit output admittance
- This is why they are called the hybrid parameters.

h parameters

- The procedure for calculating the h parameters is similar to that used \cdot for the zor y parameters.
- For reciprocal networks, $h_{12} = -h_{21}$. This can be proved in the same way .
- as we proved that $\mathbf{Z}12 = \mathbf{Z}21$. .
- · The following figure shows the hybrid model of a two-port network:

A set of parameters closely related to the h parameters are the g parameters or inverse hybrid parameters

· These are used to describe the terminal currents and voltages as:

 \cdot The values of the g parameters are determined as:

- \cdot g₁₁ = Open-circuit input admittance
- · g12 = Short-circuit reverse current gain
- · g21 = Open-circuit forward voltage gain
- · g22 = Short-circuit output impedance
- The following figure shows the inverse hybrid model of a two-۰ port network

Example 5

Find the hybrid parameters for the two-port network of the . following figure:

Example 5 Find the hybrid parameters for the two-port network of the following figure:

To find \mathbf{h}_{11} and \mathbf{h}_{21} , we short-circuit the output port and connect a current source I_1 to the input port as shown in Fig. (a) . From Fig.(a),

$$
V_1 = I_1(2 + 3 \parallel 6) = 4I_1
$$

Hence,

$$
h_{11}=\frac{V_1}{I_1}=4\ \Omega
$$

Also, from Fig.(a) we obtain, by current division,

$$
-{\bf I}_2=\frac{6}{6+3}{\bf I}_1=\frac{2}{3}{\bf I}_1
$$

 $\mathbf{h}_{21}=\frac{\mathbf{I}_2}{\mathbf{I}_1}=-\frac{2}{3}$

Hence,

Example 5 con.

To obtain h₁₂ and h₂₂, we open-circuit the input port and connect a voltage source V₂ to the output port as in Fig. (b). By voltage division,

Hence.

Also,

Thus,

TRANSMISSION PARAMETERS

 a_{11}

- The impedance and admittance parameters are grouped into the *immittance* . parameters
- The term immittance denotes a quantity that is either an impedance or an . admittance.
- The a parameters describe the voltage .
- and current at one end of the two-port
- network in term of the voltage and current ä,
- at the other end therefore they called the .
- transmission parameters
- Q11 = Open-circuit voltage ratio
- Q12 = Negative short-circuit transfer impedance
- Q22 = Open circuit transfer admittance
- Q21 = Negative short-circuit current ratio

$$
= \frac{V_1}{V_2}\bigg|_{I_2=0} \qquad a_{12} = -\frac{V_1}{I_2}\bigg|_{V_2=0} \Omega
$$

$$
a_{21} = \frac{I_1}{V_2}\bigg|_{I_2=0} S \qquad a_{22} = -\frac{I_1}{I_2}\bigg|_{V_2=0}
$$

b parameters

- The parameters b are called
- the *inverse* transmission parameters

$$
b_{11} = \frac{V_2}{V_1}\bigg|_{I_1=0} \qquad b_{12} = -\frac{V_2}{I_1}\bigg|_{V_1=0} \Omega
$$

$$
b_{21} = \frac{I_2}{V_1}\begin{vmatrix} S & b_{22} = -\frac{I_2}{I_1}\end{vmatrix}_{V_1=0}
$$

- \cdot b₁₁ = Open-circuit voltage gain
- · b12 = Negative short-circuit transfer impedance
- · b22 = Open circuit transfer admittance
- . b21 = Negative short-circuit current gain

RELATIONSHIPS BETWEEN PARAMETERS

- Since the six sets of parameters relate the same input and output \cdot terminal variables of the same two-port network, they should be interrelated.
- · If two sets of parameters exist, we can relate one set to the other set.
- \cdot Given the z parameters, let us obtain the y parameters.

or

$$
\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = [z] \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}
$$

$$
\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = [z]^{-1} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}
$$

Also, we know that:

$$
\begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \end{bmatrix} = [y] \begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \end{bmatrix}
$$

Comparing Eqs we see that

 $[y] = [z]^{-1}$

The adjoint of the [z] matrix is

$$
\begin{bmatrix} z_{22} & -z_{12} \ -z_{21} & z_{11} \end{bmatrix}
$$

and its determinant is

$$
\Delta_z = \textbf{z}_{11}\textbf{z}_{22} - \textbf{z}_{12}\textbf{z}_{21}
$$

Substituting these into Eq. [y] = $[z]^{-1}$ \cdot we get

$$
\begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} = \frac{\begin{bmatrix} z_{22} & -z_{12} \\ -z_{21} & z_{11} \end{bmatrix}}{\Delta_z}
$$

Equating terms yields

$$
y_{11} = \frac{z_{22}}{\Delta_z}
$$
, $y_{12} = -\frac{z_{12}}{\Delta_z}$, $y_{21} = -\frac{z_{21}}{\Delta_z}$, $y_{22} = \frac{z_{11}}{\Delta_z}$

As a second example, let us determine the h parameters from the z parameters. we know that .

$$
\mathbf{V}_1 = \mathbf{z}_{11}\mathbf{I}_1 + \mathbf{z}_{12}\mathbf{I}_2
$$

$$
\mathbf{V}_2 = \mathbf{z}_{21}\mathbf{I}_1 + \mathbf{z}_{22}\mathbf{I}_2
$$

Making I_2 the subject of second Eq.

$$
\mathbf{I}_2 = -\frac{\mathbf{z}_{21}}{\mathbf{z}_{22}}\mathbf{I}_1 + \frac{1}{\mathbf{z}_{22}}\mathbf{V}_2
$$

Substituting this into first Eq. .

$$
\mathbf{V}_1 = \frac{z_{11}z_{22}-z_{12}z_{21}}{z_{22}}\mathbf{I}_1 + \frac{z_{12}}{z_{22}}\mathbf{V}_2
$$

Putting Eqs in matrix form,

$$
\begin{bmatrix} \mathbf{V}_1 \\ \mathbf{I}_2 \end{bmatrix} = \begin{bmatrix} \frac{\Delta_z}{\mathbf{z}_{22}} & \frac{\mathbf{z}_{12}}{\mathbf{z}_{22}} \\ -\frac{\mathbf{z}_{21}}{\mathbf{z}_{22}} & \frac{1}{\mathbf{z}_{22}} \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{V}_2 \end{bmatrix}
$$

 \cdot For \boldsymbol{h} parameters,

$$
\begin{bmatrix} \mathbf{V}_1 \\ \mathbf{I}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{h}_{11} & \mathbf{h}_{12} \\ \mathbf{h}_{21} & \mathbf{h}_{22} \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{V}_2 \end{bmatrix}
$$

Comparing this with the last Eq., we obtain

$$
\mathbf{h}_{11} = \frac{\Delta_z}{z_{22}}, \qquad \mathbf{h}_{12} = \frac{z_{12}}{z_{22}}, \qquad \mathbf{h}_{21} = -\frac{z_{21}}{z_{22}}, \qquad \mathbf{h}_{22} = \frac{1}{z_{22}}
$$

· It can also be shown that

$$
[\mathbf{g}] = [\mathbf{h}]^{-1}
$$

· Table 18.1 provides the conversion formulas for the six sets of
two-port parameters. Given one set of parameters.

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The Contract of Street

$$
\boxed{ \begin{aligned} \mathbf{z}_{11}=\frac{\mathbf{V}_{1}}{\mathbf{I}_{1}}\bigg|_{\mathbf{I}_{2}=0}, &\mathbf{z}_{12}=\frac{\mathbf{V}_{1}}{\mathbf{I}_{2}}\bigg|_{\mathbf{I}_{1}=0} \end{aligned} \qquad \quad \boxed{ \begin{aligned} \mathbf{y}_{11}=\frac{\mathbf{I}_{1}}{\mathbf{V}_{1}}\bigg|_{\mathbf{V}_{2}=0}, &\mathbf{y}_{12}=\frac{\mathbf{I}_{1}}{\mathbf{V}_{2}}\bigg|_{\mathbf{V}_{1}=0} \end{aligned} \qquad \quad \boxed{ \begin{aligned} \mathbf{y}_{11}=\frac{\mathbf{I}_{1}}{\mathbf{V}_{1}}\bigg|_{\mathbf{V}_{2}=0}, &\mathbf{y}_{12}=\frac{\mathbf{I}_{2}}{\mathbf{V}_{2}}\bigg|_{\mathbf{V}_{1}=0} \end{aligned} \qquad \quad \boxed{ \begin{aligned} \mathbf{y}_{21}=\frac{\mathbf{I}_{2}}{\mathbf{V}_{1}}\bigg|_{\mathbf{V}_{2}=0}, &\mathbf{y}_{22}=\frac{\mathbf{I}_{2}}{\mathbf{V}_{2}}\bigg|_{\mathbf{V}_{1}=0} \end{aligned} \qquad \quad \boxed{ \begin{aligned} \mathbf{y}_{11}=\frac{\mathbf{I}_{1}}{\mathbf{V}_{1}}\bigg|_{\mathbf{V}_{2}=0}, &\mathbf{y}_{22}=\frac{\mathbf{I}_{2}}{\mathbf{V}_{2}}\bigg|_{\mathbf{V}_{1}=0} \end{aligned} \qquad \quad \boxed{ \begin{aligned} \mathbf{g}_{11}=\frac{\mathbf{I}_{1}}{\mathbf{V}_{1}}\bigg|_{\mathbf{I}_{2}=0}, &\mathbf{g}_{12}=\frac{\mathbf{I}_{1}}{\mathbf{I}_{2}}\bigg|_{\mathbf{V}_{1}=0} \end{aligned} \qquad \quad \boxed{ \begin{aligned} \mathbf{g}_{21}=\frac{\mathbf{I}_{2}}{\mathbf{V}_{1}}\bigg|_{\mathbf{I}_{2}=0}, &\mathbf{g}_{22}=\frac{\mathbf{V}_{2}}{\mathbf{I}_{2}}\bigg|_{\mathbf{V}_{1}=0} \end{aligned} \qquad \quad \boxed{ \begin{aligned} \mathbf{g}_{21}=\frac{\mathbf{V}_{2}}{\mathbf{V}_{1}}\bigg|
$$

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Note from the standpoint of the source, the load is much better matched. As a result, the effect of pulling is reduced.

However, there is a definite downside to "matching" with a fixed attenuator-the power delivered to the load is also reduced by 6 dB I

> Why keep Q: do you referring to these devices as fixed attenuators? Do you really think we would use a **broken** one?

A: In addition to fixed attenuators, engineers often used variable attenuators in radio system designs. A variable attenuator is a device whose attenuation can be adjusted (i.e., varied).

There are two types of (electronically) adjustable attenuators: digital and voltage controlled.

Digital Attenuators

As the name implies, digital attenuators are controlled with a set of digital (i.e., binary) control lines. As a result, the attenuator can be set to a specific number of discrete values.

For example, a 6-bit attenuator can be set to one of $2^6 = 64$ different attenuation values

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 $3/8$

We note therefore, that by selecting the proper switches, we can select any attenuation between 0 dB and 63 dB, in steps of $1 dB.$

For example, the 6-bit binary word 101101 would result in attenuation of:

$$
32 + 8 + 4 + 1 = 45
$$
 dB

Note also that 101101 is the binary representation of the decimal number 45-the binary control word equals the attenuation in dBII

Voltage Controlled Attenuators

Another adjustable attenuator is the voltage-controlled attenuator. This device uses a single control line, with the voltage at that control determining the attenuation of the device (an "analog" attenuatorl):

Attenuation = $f(V_c)$

$$
\frac{1}{\frac{P_{in}}{P_{in}}}
$$

Typical voltage control attenuators can provide attenuation from a minimum of a few dB to a maximum of as much as 50 dB.

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Unlike the digital attenuator, this attenuation range is a continuous function of V_c , so that any and every attenuation between the minimum and maximum values can be selected.

Voltage controlled attenuators are typically smaller, simpler, and cheaper than their digital counterparts.

Q: So why did you waste our time with digital attenuators? It sounds like voltage controlled attenuators are always the way to go!

A: We have yet to discuss the bad stuff about voltage controlled attenuatorsL

* Voltage controlled attenuators are generally speaking poorly matched, with a return loss that varies with the control voltage V_c

* Likewise, the phase delay, bandwidth, and just about every other device parameter also changes with Vcl

* Moreover, voltage controlled attenuators are notoriously sensitive to temperature, power supply variations, and load impedance.

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UNIT-04

Network Functions for Simple Circuits

Introduction

Each of the circuits in this problem set is represented by a network function. Network functions are defined, in the frequency-domain, to be quotient obtained by dividing the phasor corresponding to the circuit output by the phasor corresponding to the circuit input. We calculate the network function of a circuit by representing and analyzing the circuit in the frequencydomain.

Network functions are described in Section 13.3 of Introduction to Electric Circuits by R.C. Dorf and J.A Svoboda. Also, Table 10.7-1 summarizes the correspondence between the timedomain and the frequency domain.

Worked Examples

Example 1:

Consider the circuit shown in Figure 1. The input to the circuit is the voltage of the voltage source, $v_i(t)$. The output is the voltage across the 8 Ω resistor, $v_o(t)$. The network function that represents this circuit is

$$
\mathbf{H}(\omega) = \frac{\mathbf{V_o}(\omega)}{\mathbf{V_i}(\omega)} = \frac{0.66}{1 + j\frac{\omega}{30}}
$$
(1)

Determine the value of the inductance, L.

Figure 1 The circuit considered in Example 1.

Solution: The circuit has been represented twice, by a circuit diagram and also by a network function. The unknown inductance, L, appears in the circuit diagram, but not in the given network function. We can analyze the circuit to determine its network function. This second network function will depend on the unknown inductance. We will determine the value of the inductance by equating the two network functions.

A network function is the ratio of the output phasor to the input phasor. Phasors exist in the frequency domain. Consequently, our first step is to represent the circuit in the frequency domain, using phasors and impedances. Figure 2 shows the frequency domain representation of the circuit from Figure 1.

Figure 2 The circuit from Figure 1, represented in the frequency domain, using impedances and phasors.

The impedances of the inductor and the two resistors are connected in series in Figure 2. $V_i(\omega)$ is the voltage across these three series impedances and $V_0(\omega)$ is the voltage across one of the impedances. Apply the voltage division principle to get

$$
\mathbf{V}_{\mathbf{o}}\left(\omega\right) = \frac{8}{4 + 8 + j\,\omega\,L}\,\mathbf{V}_{\mathbf{i}}\left(\omega\right)
$$

Divide both sides of this equation by $V_i(\omega)$ to obtain the network function of the circuit

$$
\mathbf{H}(\omega) = \frac{\mathbf{V}_0(\omega)}{\mathbf{V}_i(\omega)} = \frac{8}{12 + j\omega L}
$$
 (2)

The network functions given in Equations 1 and 2 must be equal. That is

$$
\frac{8}{12 + j\omega L} = \frac{0.66}{1 + j\frac{\omega}{30}}
$$

\n
$$
8\left(1 + j\frac{\omega}{30}\right) = 0.66(12 + j\omega L)
$$

\n
$$
8 + j\frac{8\omega}{30} = 8 + j(0.66)\omega L
$$

\n
$$
\frac{8}{30} = (0.66)L
$$

\n
$$
L = \frac{8}{30(0.66)}
$$

\n
$$
L = 0.4 \text{ H}
$$

We can simply the algebra required to find L by putting the network function in Equation 2 into the same form as the network function in Equation 1 before equating the two network functions. Notice that the real part of the denominator of the network function is 1 in Equation 1. Let's make the real part of the denominator be 1 in the network function given by Equation 2. Divide the numerator and denominator by 12 to get

$$
\mathbf{H}(\omega) = \frac{\mathbf{V}_{\mathbf{o}}(\omega)}{\mathbf{V}_{\mathbf{i}}(\omega)} = \frac{\frac{8}{12}}{\frac{12 + j\omega L}{12}} = \frac{0.66}{1 + j\omega \frac{L}{12}}
$$
(3)

Equating the network functions given by Equations 1 and 3 gives:

$$
\frac{0.66}{1+j\omega}\frac{L}{12} = \frac{0.66}{1+j\frac{\omega}{30}} \Rightarrow \frac{L}{12} = \frac{1}{30} \Rightarrow L = 0.4 \text{ H}
$$

The same result is obtained with less algebra.

Example 2:

Consider the circuit shown in Figure 3. The input to the circuit is the voltage of the voltage source, $v_i(t)$. The output is the voltage across the 4 Ω resistor, $v_o(t)$. This circuit is an example of a "first order low-pass filter". The network function that represents a first order low-pass filter has the form

$$
\mathbf{H}(\omega) = \frac{\mathbf{V}_{\mathbf{o}}(\omega)}{\mathbf{V}_{\mathbf{i}}(\omega)} = \frac{k}{1 + j\frac{\omega}{p}}
$$
(4)

This network function depends on two parameters, k and p . The parameter k is called the "dc gain" of the first order low-pass filter and p is the pole of the first order low-pass filter. Determine the values of k and of p for the first order low-pass filter in Figure 3.

Figure 3 The circuit considered in Example 2.

Solution: We will analyze the circuit to determine its network function and then put the network function into the form given in Equation 4. A network function is the ratio of the output phasor to the input phasor. Phasors exist in the frequency domain. Consequently, our first step is to represent the circuit in the frequency domain, using phasors and impedances. Figure 4 shows the frequency domain representation of the circuit from Figure 3.

Figure 4 The circuit from Figure 3, represented in the frequency domain, using impedances and phasors.

The impedances of the inductor and the two resistors are connected in series in Figure 4. $V_i(\omega)$ is the voltage across these three series impedances and $V_o(\omega)$ is the voltage across one of the impedances. Apply the voltage division principle to get

$$
\mathbf{V}_{\mathbf{o}}\left(\omega\right) = \frac{4}{9+4+j\left(0.25\right)\omega}\mathbf{V}_{\mathbf{i}}\left(\omega\right)
$$

Divide both sides of this equation by $V_i(\omega)$ to obtain the network function of the circuit

$$
\mathbf{H}\left(\omega\right) = \frac{\mathbf{V}_{\mathbf{a}}\left(\omega\right)}{\mathbf{V}_{\mathbf{i}}\left(\omega\right)} = \frac{4}{13 + j\left(0.25\right)\omega} \tag{5}
$$

Next, we put the network function into the form specified by Equation 4. Notice that the real part of the denominator is 1 in Equation 4. Divide the numerator and denominator by 13 in Equation 5 to get

$$
\mathbf{H}(\omega) = \frac{\mathbf{V}_{\mathbf{o}}(\omega)}{\mathbf{V}_{\mathbf{i}}(\omega)} = \frac{\frac{4}{13}}{\frac{13}{13} + \frac{j(0.25)\omega}{13}} = \frac{0.308}{1 + \frac{j\omega}{52}}
$$
(6)

Comparing the network functions given by Equations 4 and 6 gives

$$
k = 0.308
$$
 V/V and $p = 52$ rad/s

Example 3:

Consider the circuit shown in Figure 5. The input to the circuit is the voltage of the voltage source, $v_i(t)$. The output is the voltage across the 5 Ω resistor, $v_o(t)$. The network function that represents this circuit is

$$
\mathbf{H}\left(\omega\right) = \frac{\mathbf{V_o}\left(\omega\right)}{\mathbf{V_i}\left(\omega\right)} = 0.208 \frac{j\omega}{1 + j\frac{\omega}{2}}\tag{7}
$$

Determine the value of the capacitance, C.

Figure 5 The circuit considered in Example 3.

Solution: The circuit has been represented twice, by a circuit diagram and also by a network function. The unknown capacitance, C, appears in the circuit diagram, but not in the given network function. We can analyze the circuit to determine its network function. This second network function will depend on the unknown capacitance. We will determine the value of the capacitance by equating the two network functions.

A network function is the ratio of the output phasor to the input phasor. Phasors exist in the frequency domain. Consequently, our first step is to represent the circuit in the frequency domain, using phasors and impedances. Figure 6 shows the frequency domain representation of the circuit from Figure 5.

Figure 6 The circuit from Figure 5, represented in the frequency domain, using impedances and phasors.

The impedances of the capacitor and the two resistors are connected in series in Figure 6. $V_i(\omega)$ is the voltage across these three series impedances and $V_0(\omega)$ is the voltage across one of the impedances. Apply the voltage division principle to get

$$
\mathbf{V}_{\mathbf{a}}(\omega) = \frac{5}{5+3+\frac{1}{j\omega C}}\mathbf{V}_{\mathbf{i}}(\omega)
$$

Divide both sides of this equation by $V_i(\omega)$ to obtain the network function of the circuit

$$
\mathbf{H}(\omega) = \frac{\mathbf{V_o}(\omega)}{\mathbf{V_i}(\omega)} = \frac{5}{5 + 3 + \frac{1}{j\omega C}}
$$
(8)

We can simply the algebra required to find C by putting the network function in Equation 8 into the same form as the network function in Equation 7 before equating the two network functions. Let's multiply the numerator and denominator by $j\omega C$ to get

$$
\mathbf{H}(\omega) = \frac{\mathbf{V}_0(\omega)}{\mathbf{V}_1(\omega)} = \frac{5}{8 + \frac{1}{j\omega C}} \times \frac{j\omega C}{j\omega C} = 5C \frac{j\omega}{1 + j\omega C(8)}\tag{9}
$$

Equating the network functions given by Equations 7 and 9 gives:

$$
5C \frac{j\omega}{1+j\omega C(8)} = 0.208 \frac{j\omega}{1+j\frac{\omega}{3}}
$$

Comparing corresponding parts of this equation indicates that:

$$
5C = 0.208
$$
 and $8C = \frac{1}{3}$

The values of C obtained from these equations must agree. (If they do not, we've made an error.) Solving these equations gives

$$
C = 41.60
$$
 mF and $C = 41.67$ mF

These values agree, but there is some uncertainty in the third significant figure. It's appropriate to report our result with two significant figures:

$$
C = 42 \text{ mF}
$$

Example 4:

Consider the circuit shown in Figure 7. The input to the circuit is the voltage of the voltage source, $v_1(t)$. The output is the voltage across the 8 Ω resistor, $v_2(t)$. This circuit is an example of a "first order high-pass filter". The network function that represents a first order high-pass filter has the form

$$
\mathbf{H}\left(\omega\right) = \frac{\mathbf{V}_{\mathbf{o}}\left(\omega\right)}{\mathbf{V}_{\mathbf{i}}\left(\omega\right)} = k \frac{j\omega}{1 + j\frac{\omega}{p}}
$$
(10)

The network function depends on two parameters, k and p . The parameter p is called the pole of the first order high-pass filter. The parameter k is sometime referred to as a gain, but the highfrequency gain of the circuit is given by the product kp . Determine the values of k and of p for the first order high-pass filter in Figure 7.

Figure 7 The circuit considered in Example 4.

Solution: We will analyze the circuit to determine its network function and then put the network function into the form given in Equation 10. A network function is the ratio of the output phasor to the input phasor. Phasors exist in the frequency domain. Consequently, our first step is to represent the circuit in the frequency domain, using phasors and impedances. Figure 8 shows the frequency domain representation of the circuit from Figure 7.

Figure 8 The circuit from Figure 7, represented in the frequency domain, using impedances and phasors.

The impedances of the inductor and one 8Ω resistor are connected in parallel in Figure 8. The equivalent impedance is
$$
\mathbf{Z}_*(\omega) = \frac{(8)\,j\,(0.8)\,\omega}{(8) + j\,(0.8)\,\omega}
$$

The parallel impedance is connected in series with the other 8 Ω resistor. $V_i(\omega)$ is the voltage across the series impedances and $V_o(\omega)$ is the voltage across the equivalent impedance, $Z_e(\omega)$. Apply the voltage division principle to get

$$
\mathbf{V}_{\mathbf{0}}(\omega) = \frac{\frac{(8) j(0.8)\omega}{(8) + j(0.8)\omega}}{8 + \frac{(8) j(0.8)\omega}{(8) + j(0.8)\omega}} \mathbf{V}_{\mathbf{i}}(\omega)
$$

=
$$
\frac{(8) j(0.8)\omega}{8((8) + j(0.8)\omega) + (8) j(0.8)\omega} \mathbf{V}_{\mathbf{i}}(\omega)
$$

=
$$
\frac{j(6.4)\omega}{64 + j(8)(2)(0.8)\omega} \mathbf{V}_{\mathbf{i}}(\omega)
$$

=
$$
\frac{j(6.4)\omega}{64 + j(12.8)\omega} \mathbf{V}_{\mathbf{i}}(\omega)
$$

Divide both sides of this equation by $V_i(\omega)$ to obtain the network function of the circuit

$$
\mathbf{H}\left(\omega\right) = \frac{\mathbf{V_o}\left(\omega\right)}{\mathbf{V_i}\left(\omega\right)} = \frac{j\left(6.4\right)\omega}{64 + j\left(12.8\right)\omega} \tag{11}
$$

Next, we put the network function into the form specified by Equation 10. Notice that the real part of the denominator is 1 in Equation 10. Divide the numerator and denominator by 64 in Equation 11 to get

$$
\mathbf{H}\left(\omega\right) = \frac{\mathbf{V}_{\mathbf{o}}\left(\omega\right)}{\mathbf{V}_{\mathbf{i}}\left(\omega\right)} = \frac{\frac{j(6.4)\omega}{64}}{\frac{64 + j(12.8)\omega}{64}} = 0.1 \frac{j\omega}{1 + \frac{j(12.8)\omega}{64}}\tag{12}
$$

Comparing the network functions given by Equations 10 and 12 gives

$$
k = 0.1
$$
 V/V and $p = \frac{64}{12.8} = 5$ rad/s.

Example 5:

Consider the circuit shown in Figure 9. The input to the circuit is the voltage of the voltage source, $v_i(t)$. The output is the voltage, $v_o(t)$, across the series connection of the capacitor and 16 $k\Omega$ resistor. The network function that represents a this circuit has the form

$$
\mathbf{H}\left(\omega\right) = \frac{\mathbf{V}_{\mathbf{a}}\left(\omega\right)}{\mathbf{V}_{\mathbf{i}}\left(\omega\right)} = \frac{1 + j\frac{\omega}{z}}{1 + j\frac{\omega}{\rho}}
$$
(13)

63

The network function depends on two parameters, z and p . The parameter z is called the zero of the circuit and the parameter p is called the pole of the circuit. Determine the values of z and of p for the circuit in Figure 9.

Figure 9 The circuit considered in Example 5.

Solution: We will analyze the circuit to determine its network function and then put the network function into the form given in Equation 13. A network function is the ratio of the output phasor to the input phasor. Phasors exist in the frequency domain. Consequently, our first step is to represent the circuit in the frequency domain, using phasors and impedances. Figure 10 shows the frequency domain representation of the circuit from Figure 9.

Figure 10 The circuit from Figure 9, represented in the frequency domain, using impedances and phasors.

The impedances of the capacitor and the 16 k Ω resistor are connected in series in Figure 10. The equivalent impedance is

$$
\mathbf{Z}_{*}(\omega) = 16000 + \frac{10^6}{j(0.23)\omega}
$$

The equivalent impedance is connected in series with the 8 k Ω resistor. $V_i(\omega)$ is the voltage across the series impedances and $V_{\theta}(\omega)$ is the voltage across the equivalent impedance, $Z_{\epsilon}(\omega)$. Apply the voltage division principle to get

$$
\mathbf{V}_{\mathbf{o}}(\omega) = \frac{16000 + \frac{10^6}{j(0.23)\omega}}{8000 + 16000 + \frac{10^6}{j(0.23)\omega}} \mathbf{V}_{\mathbf{i}}(\omega)
$$

$$
= \frac{10^6 + j(0.23)\omega(16000)}{10^6 + j(0.23)\omega(24000)} \mathbf{V}_{\mathbf{i}}(\omega)
$$

$$
= \frac{\frac{10^6 + j(3680)\omega}{10^6}}{\frac{10^6}{10^6}} \mathbf{V}_{\mathbf{i}}(\omega)
$$

$$
= \frac{1 + j(0.00368)\omega}{1 + j(0.00552)\omega} \mathbf{V}_{\mathbf{i}}(\omega)
$$

Divide both sides of this equation by $V_i(\omega)$ to obtain the network function of the circuit

$$
\mathbf{H}\left(\omega\right) = \frac{\mathbf{V}_{\mathbf{0}}\left(\omega\right)}{\mathbf{V}_{\mathbf{i}}\left(\omega\right)} = \frac{1 + j\left(0.00368\right)\omega}{1 + j\left(0.00552\right)\omega} \tag{14}
$$

Equating the network functions given by Equations 13 and 14 gives

$$
\frac{1+j(0.00368)\omega}{1+j(0.00552)\omega} = \frac{1+j\frac{\omega}{z}}{1+j\frac{\omega}{p}}
$$

Comparing these network functions gives

$$
z = \frac{1}{0.00368} = 271.74 \text{ rad/s and } p = \frac{1}{0.00552} = 181.16 \text{ rad/s}.
$$

Example 6:

Consider the circuit shown in Figure 11. The input to the circuit is the voltage of the voltage source, $v_i(t)$. The output is the voltage, $v_0(t)$, across series connection of the inductor the 2 Ω resistor. The network function that represents this circuit is

$$
\mathbf{H}\left(\omega\right) = \frac{\mathbf{V}_{\mathbf{o}}\left(\omega\right)}{\mathbf{V}_{\mathbf{i}}\left(\omega\right)} = 0.2 \frac{1 + j\frac{\omega}{5}}{1 + j\frac{\omega}{25}}\tag{15}
$$

Determine the value of the inductance, L.

Figure 11 The circuit considered in Example 6.

Solution: The circuit has been represented twice, by a circuit diagram and also by a network function. The unknown inductance, L, appears in the circuit diagram, but not in the given network function. We can analyze the circuit to determine its network function. This second network function will depend on the unknown inductance. We will determine the value of the inductance by equating the two network functions.

A network function is the ratio of the output phasor to the input phasor. Phasors exist in the frequency domain. Consequently, our first step is to represent the circuit in the frequency domain, using phasors and impedances. Figure 12 shows the frequency domain representation of the circuit from Figure 11.

Figure 12 The circuit from Figure 11, represented in the frequency domain, using impedances and phasors.

The impedances of the inductor and the 2Ω resistor are connected in series in Figure 12. The equivalent impedance is

$$
Z_{\bullet}(\omega) = 2 + j \omega L
$$

The equivalent impedance is connected in series with the 8 Ω resistor. $V_i(\omega)$ is the voltage across the series impedances and $V_0(\omega)$ is the voltage across the equivalent impedance, $Z_e(\omega)$. Apply the voltage division principle to get

$$
\mathbf{V}_{\mathbf{o}}(\omega) = \frac{2 + j\omega L}{8 + 2 + j\omega L} \mathbf{V}_{\mathbf{i}}(\omega) = \frac{2 + j\omega L}{10 + j\omega L} \mathbf{V}_{\mathbf{i}}(\omega)
$$

Divide both sides of this equation by $V_i(\omega)$ to obtain the network function of the circuit

$$
\mathbf{H}\left(\omega\right) = \frac{\mathbf{V}_{\mathbf{0}}\left(\omega\right)}{\mathbf{V}_{\mathbf{i}}\left(\omega\right)} = \frac{2 + j\,\omega\,L}{10 + j\,\omega\,L}
$$

Next, we put the network function into the form specified by Equation 15. Factoring 2 out of both terms in the numerator and also factoring 10 out of both terms in the denominator we get

$$
\mathbf{H}(\omega) = \frac{2\left(1+j\omega\frac{L}{2}\right)}{10\left(1+j\omega\frac{L}{10}\right)} = 0.2\frac{1+j\omega\frac{L}{2}}{1+j\omega\frac{L}{10}}
$$
(16)

Equating the network functions given by Equations 15 and 16 gives

$$
0.2 \frac{1+j\omega\frac{L}{2}}{1+j\omega\frac{L}{10}} = 0.2 \frac{1+j\frac{\omega}{5}}{1+j\frac{\omega}{25}}
$$

Comparing these network functions gives

$$
\frac{L}{2} = \frac{1}{5}
$$
 and $\frac{L}{10} = \frac{1}{25}$

The values of L obtained from these equations must agree, and they do. (If they do not, we've made an error.) Solving each of these equations gives $L = 0.4$ H.

The impedances of the inductor and the 2Ω resistor are connected in series in Figure 12. The equivalent impedance is

$$
Z_{\bullet}(\omega) = 2 + j \omega L
$$

The equivalent impedance is connected in series with the 8 Ω resistor. $V_i(\omega)$ is the voltage across the series impedances and $V_0(\omega)$ is the voltage across the equivalent impedance, $Z_e(\omega)$. Apply the voltage division principle to get

$$
\mathbf{V}_{\mathbf{o}}(\omega) = \frac{2 + j\omega L}{8 + 2 + j\omega L} \mathbf{V}_{\mathbf{i}}(\omega) = \frac{2 + j\omega L}{10 + j\omega L} \mathbf{V}_{\mathbf{i}}(\omega)
$$

Divide both sides of this equation by $V_i(\omega)$ to obtain the network function of the circuit

$$
\mathbf{H}\left(\omega\right) = \frac{\mathbf{V}_{\mathbf{0}}\left(\omega\right)}{\mathbf{V}_{\mathbf{i}}\left(\omega\right)} = \frac{2 + j\,\omega\,L}{10 + j\,\omega\,L}
$$

Next, we put the network function into the form specified by Equation 15. Factoring 2 out of both terms in the numerator and also factoring 10 out of both terms in the denominator we get

$$
\mathbf{H}(\omega) = \frac{2\left(1+j\omega\frac{L}{2}\right)}{10\left(1+j\omega\frac{L}{10}\right)} = 0.2\frac{1+j\omega\frac{L}{2}}{1+j\omega\frac{L}{10}}
$$
(16)

Equating the network functions given by Equations 15 and 16 gives

$$
0.2 \frac{1+j\omega\frac{L}{2}}{1+j\omega\frac{L}{10}} = 0.2 \frac{1+j\frac{\omega}{5}}{1+j\frac{\omega}{25}}
$$

Comparing these network functions gives

$$
\frac{L}{2} = \frac{1}{5}
$$
 and $\frac{L}{10} = \frac{1}{25}$

The values of L obtained from these equations must agree, and they do. (If they do not, we've made an error.) Solving each of these equations gives $L = 0.4$ H.

domain, using phasors and impedances. Figure 14 shows the frequency domain representation of the circuit from Figure 13.

The impedances of the capacitor and the $4 k\Omega$ resistor are connected in series in Figure 14. The equivalent impedance is

$$
\mathbf{Z}_{e}(\omega) = 4000 + \frac{1}{j\omega C}
$$

The equivalent impedance is connected in series with the 1 k Ω resistor. $V_i(\omega)$ is the voltage across the series impedances and $V_o(\omega)$ is the voltage across the equivalent impedance, $Z_e(\omega)$. Apply the voltage division principle to get

$$
\mathbf{V}_{\mathbf{o}}(\omega) = \frac{4000 + \frac{1}{j\omega C}}{1000 + 4000 + \frac{1}{j\omega C}} \mathbf{V}_{\mathbf{i}}(\omega) = \frac{1 + j\omega C(4000)}{1 + j\omega C(5000)} \mathbf{V}_{\mathbf{i}}(\omega)
$$

Divide both sides of this equation by $V_i(\omega)$ to obtain the network function of the circuit

l,

$$
\mathbf{H}(\omega) = \frac{\mathbf{V_o}(\omega)}{\mathbf{V_i}(\omega)} = \frac{1 + j\omega C (4000)}{1 + j\omega C (5000)}\tag{18}
$$

Equating the network functions given by Equations 17 and 18 gives

$$
\frac{1+j\omega C(4000)}{1+j\omega C(5000)} = \frac{1+j\frac{\omega}{69.6}}{1+j\frac{\omega}{55.7}}
$$

Comparing these network functions gives

$$
4000 C = \frac{1}{69.9}
$$
 and $5000 C = \frac{1}{55.7}$

The values of C obtained from these equations must agree, and they do. (If they do not, we've made an error.) Solving these equations gives

$$
C = 3.577 \text{ }\mu\text{F}
$$
 and $C = 3.591 \text{ }\mu\text{F}$

These values agree, but there is some uncertainty in the third significant figure. It's appropriate to report our result with two significant figures:

$$
C = 3.6 \text{ }\mu\text{F}
$$

UNIT-05

The Fourier Transform

Fourier transforms as integrals 1.1

There are several ways to define the Fourier transform of a function $f : \mathbb{R} \to$ C. In this section, we define it using an integral representation and state some basic uniqueness and inversion properties, without proof. Thereafter, we will consider the transform as being defined as a suitable limit of Fourier series, and will prove the results stated here.

Definition 1 Let $f : \mathbb{R} \to \mathbb{R}$. The Fourier transform of $f \in L^1(\mathbb{R})$, denoted by $\mathcal{F}[f](.)$, is given by the integral:

$$
\mathcal{F}[f](x) := \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(t) \exp(-ixt) dt
$$

for $x \in \mathbb{R}$ for which the integral exists. \degree

We have the Dirichlet condition for inversion of Fourier integrals.

Theorem 1 Let $f : \mathbb{R} \to \mathbb{R}$. Suppose that (1) $\int_{-\infty}^{\infty} |f| dt$ converges and (2) in any finite interval, f, f' are piecewise continuous with at most finitely many maxima/minima/discontinuities. Let $F = \mathcal{F}[f]$. Then if f is continuous at $t \in \mathbb{R}$, we have

$$
f(t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} F(x) \exp(itx) dx.
$$

^{*}This definition also makes sense for complex valued f but we stick here to real valued \mathbf{r}

Moreover, if f is discontinuous at $t \in \mathbb{R}$ and $f(t+0)$ and $f(t-0)$ denote the right and left limits of f at t, then

$$
\frac{1}{2}[f(t+0)+f(t-0)]=\frac{1}{\sqrt{2\pi}}\int_{\infty}^{\infty}F(x)\exp(itx)dx.
$$

From the above, we deduce a uniqueness result:

Theorem 2 Let $f, g : \mathbb{R} \to \mathbb{R}$ be continuous, f', g' piecewise continuous. If

$$
\mathcal{F}[f](x) = \mathcal{F}[g](x), \forall x
$$

 $then$

$$
f(t) = g(t), \forall t.
$$

Proof: We have from inversion, easily that

$$
f(t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \mathcal{F}[f](x) \exp(itx) dx
$$

=
$$
\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \mathcal{F}[g](x) \exp(itx) dx
$$

=
$$
g(t).
$$

 \Box

Example 1 Find the Fourier transform of $f(t) = \exp(-|t|)$ and hence using inversion, deduce that $\int_0^\infty \frac{dx}{1+x^2} = \frac{\pi}{2}$ and $\int_0^\infty \frac{r \sin(xt)}{1+x^2} dx = \frac{\pi \exp(-t)}{2}$, $t > 0$.

Solution We write

$$
F(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(t) \exp(-ixt) dt
$$

= $\frac{1}{\sqrt{2\pi}} \left[\int_{-\infty}^{0} \exp(t(1 - ix)) dt + \int_{0}^{\infty} \exp(-t(1 + ix)) \right]$
= $\sqrt{\frac{2}{\pi}} \frac{1}{1 + x^2}.$

Now by the inversion formula,

$$
\exp(-|t|) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} F(x) \exp(ixt) dx
$$

$$
= \frac{1}{\pi} \left[\int_{0}^{\infty} \frac{\exp(ixt) + \exp(-ixt)}{1 + x^2} dt \right]
$$

$$
= \frac{2}{\pi} \int_{0}^{\infty} \frac{\cos(xt)}{1 + x^2} dx.
$$

 $\,2$

Now this formula holds at $t = 0$, so substituting $t = 0$ into the above gives the first required identity. Differentiating with respect to t as we may for $t > 0$, gives the second required identity. \Box .

Proceeding in a similar way as the above example, we can easily show that h an

$$
\mathcal{F}[\exp(-\frac{1}{2}t^2)](x) = \exp(-\frac{1}{2}x^2), x \in \mathbb{R}.
$$

We will discuss this example in more detail later in this chapter.

We will also show that we can reinterpret Definition 1 to obtain the Fourier transform of any complex valued $f \in L^2(\mathbb{R})$, and that the Fourier transform is unitary on this space:

Theorem 3 If $f, g \in L^2(\mathbb{R})$ then $\mathcal{F}[f], \mathcal{F}[g] \in L^2(\mathbb{R})$ and

$$
\int_{-\infty}^{\infty} f(t)\overline{g}(t) dt = \int_{-\infty}^{\infty} \mathcal{F}[f](x)\overline{\mathcal{F}[g](x)} dx.
$$

This is a result of fundamental importance for applications in signal processing.

1.2 The transform as a limit of Fourier series

We start by constructing the Fourier series (complex form) for functions on an interval $[-\pi L, \pi L]$. The ON basis functions are

$$
e_n(t) = \frac{1}{\sqrt{2\pi L}} e^{\frac{int}{L}}, \qquad n = 0, \pm 1, \cdots,
$$

and a sufficiently smooth function f of period $2\pi L$ can be expanded as

$$
f(t) = \sum_{n=-\infty}^{\infty} \left(\frac{1}{2\pi L} \int_{-\pi L}^{\pi L} f(x) e^{-\frac{i\pi x}{L}} dx \right) e^{\frac{i\pi t}{L}}.
$$

For purposes of motivation let us abandon periodicity and think of the functions f as differentiable everywhere, vanishing at $t = \pm \pi L$ and identically zero outside $[-\pi L, \pi L]$. We rewrite this as

$$
f(t) = \sum_{n=-\infty}^{\infty} e^{\frac{int}{L}} \frac{1}{2\pi L} \hat{f}(\frac{n}{L})
$$

which looks like a Riemann sum approximation to the integral

$$
f(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \hat{f}(\lambda) e^{i\lambda t} d\lambda \qquad (1.2.1)
$$

 $\sqrt{3}$
to which it would converge as $L \to \infty$. (Indeed, we are partitioning the λ interval $[-L, L]$ into 2L subintervals, each with partition width $1/L$.) Here,

$$
\hat{f}(\lambda) = \int_{-\infty}^{\infty} f(t)e^{-i\lambda t}dt.
$$
\n(1.2.2)

Similarly the Parseval formula for f on $[-\pi L, \pi L]$,

$$
\int_{-\pi L}^{\pi L} |f(t)|^2 dt = \sum_{n=-\infty}^{\infty} \frac{1}{2\pi L} |\hat{f}(\frac{n}{L})|^2
$$

goes in the limit as $L \to \infty$ to the *Plancherel identity*

$$
2\pi \int_{-\infty}^{\infty} |f(t)|^2 dt = \int_{-\infty}^{\infty} |\hat{f}(\lambda)|^2 d\lambda.
$$
 (1.2.3)

Expression (1.2.2) is called the Fourier integral or Fourier transform of f. Expression $(1.2.1)$ is called the *inverse Fourier integral* for f . The Plancherel identity suggests that the Fourier transform is a one-to-one norm preserving map of the Hilbert space $L^2[-\infty,\infty]$ onto itself (or to another copy of itself). We shall show that this is the case. Furthermore we shall show that the pointwise convergence properties of the inverse Fourier transform are somewhat similar to those of the Fourier series. Although we could make a rigorous justification of the the steps in the Riemann sum approximation above, we will follow a different course and treat the convergence in the mean and pointwise convergence issues separately.

A second notation that we shall use is

$$
\mathcal{F}[f](\lambda) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(t)e^{-i\lambda t}dt = \frac{1}{\sqrt{2\pi}}\hat{f}(\lambda)
$$
 (1.2.4)

$$
\mathcal{F}^*[g](t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} g(\lambda) e^{i\lambda t} d\lambda \qquad (1.2.5)
$$

Note that, formally, $\mathcal{F}^*[\hat{f}](t) = \sqrt{2\pi} f(t)$. The first notation is used more often in the engineering literature. The second notation makes clear that $\mathcal F$ and \mathcal{F}^* are linear operators mapping $L^2[-\infty,\infty]$ onto itself in one view, and $\mathcal F$ mapping the *signal space* onto the *frequency space* with $\mathcal F^*$ mapping the frequency space onto the signal space in the other view. In this notation the Plancherel theorem takes the more symmetric form

$$
\int_{-\infty}^{\infty} |f(t)|^2 dt = \int_{-\infty}^{\infty} |\mathcal{F}[f](\lambda)|^2 d\lambda.
$$

Examples:

 $\overline{4}$

1. The box function (or rectangular wave)

$$
\Pi(t) = \begin{cases} 1 & \text{if } -\pi < t < \pi \\ \frac{1}{2} & \text{if } t = \pm \pi \\ 0 & \text{otherwise.} \end{cases} \tag{1.2.6}
$$

Then, since $\Pi(t)$ is an even function and $e^{-i\lambda t} = \cos(\lambda t) + i \sin(\lambda t)$, we have

$$
\hat{\Pi}(\lambda) = \sqrt{2\pi} \mathcal{F}[\Pi](\lambda) = \int_{-\infty}^{\infty} \Pi(t)e^{-i\lambda t}dt = \int_{-\infty}^{\infty} \Pi(t)\cos(\lambda t)dt
$$

$$
= \int_{-\pi}^{\pi} \cos(\lambda t)dt = \frac{2\sin(\pi\lambda)}{\lambda} = 2\pi \text{ sinc }\lambda.
$$

Thus sinc λ is the Fourier transform of the box function. The inverse Fourier transform is

$$
\int_{-\infty}^{\infty} \operatorname{sinc}(\lambda) e^{i\lambda t} d\lambda = \Pi(t),\tag{1.2.7}
$$

as follows from (??). Furthermore, we have

$$
\int_{-\infty}^{\infty} |\Pi(t)|^2 dt = 2\pi
$$

and

$$
\int_{-\infty}^{\infty}|\ {\rm sinc}\ (\lambda)|^2 d\lambda=1
$$

from (??), so the Plancherel equality is verified in this case. Note that the inverse Fourier transform converged to the midpoint of the discontinuity, just as for Fourier series.

2. A truncated cosine wave.

$$
f(t) = \begin{cases} \cos 3t & \text{if } -\pi < t < \pi \\ -\frac{1}{2} & \text{if } t = \pm \pi \\ 0 & \text{otherwise.} \end{cases}
$$

Then, since the cosine is an even function, we have

$$
\hat{f}(\lambda) = \sqrt{2\pi} \mathcal{F}[f](\lambda) = \int_{-\infty}^{\infty} f(t)e^{-i\lambda t}dt = \int_{-\pi}^{\pi} \cos(3t)\cos(\lambda t)dt
$$

$$
= \frac{2\lambda\sin(\lambda\pi)}{9-\lambda^2}.
$$

 $5\,$

3. A truncated sine wave.

$$
f(t) = \begin{cases} \sin 3t & \text{if } -\pi \le t \le \pi \\ 0 & \text{otherwise.} \end{cases}
$$

Since the sine is an odd function, we have

$$
\hat{f}(\lambda) = \sqrt{2\pi} \mathcal{F}[f](\lambda) = \int_{-\infty}^{\infty} f(t)e^{-i\lambda t}dt = -i \int_{-\pi}^{\pi} \sin(3t)\sin(\lambda t)dt
$$

$$
= \frac{-6i\sin(\lambda\pi)}{9-\lambda^2}.
$$

4. A triangular wave.

$$
f(t) = \begin{cases} 1+t & \text{if } -1 \le t \le 0 \\ -1 & \text{if } 0 \le t \le 1 \\ 0 & \text{otherwise.} \end{cases} \tag{1.2.8}
$$

Then, since f is an even function, we have

$$
\hat{f}(\lambda) = \sqrt{2\pi} \mathcal{F}[f](\lambda) = \int_{-\infty}^{\infty} f(t)e^{-i\lambda t}dt = 2\int_{0}^{1} (1-t)\cos(\lambda t)dt
$$

$$
= \frac{2 - 2\cos\lambda}{\lambda^2}.
$$

NOTE: The Fourier transforms of the discontinuous functions above decay as $\frac{1}{\lambda}$ for $|\lambda| \to \infty$ whereas the Fourier transforms of the continuous functions
decay as $\frac{1}{\lambda^2}$. The coefficients in the Fourier series of the analogous functions
decay as $\frac{1}{n}$, $\frac{1}{n^2}$, respectivel

$1.2.1$ Properties of the Fourier transform

Recall that

$$
\mathcal{F}[f](\lambda) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(t)e^{-i\lambda t}dt = \frac{1}{\sqrt{2\pi}}\hat{f}(\lambda)
$$

$$
\mathcal{F}^*[g](t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} g(\lambda)e^{i\lambda t}d\lambda
$$

We list some properties of the Fourier transform that will enable us to build a repertoire of transforms from a few basic examples. Suppose that f, g belong to $L^1[-\infty, \infty]$, i.e., $\int_{-\infty}^{\infty} |f(t)|dt < \infty$ with a similar statement for *g*. We can state the following (whose straightforward proofs are left to the reader):

 66

1. $\mathcal F$ and $\mathcal F^*$ are linear operators. For $a, b \in C$ we have

$$
\mathcal{F}[af + bg] = a\mathcal{F}[f] + b\mathcal{F}[g], \quad \mathcal{F}^*[af + bg] = a\mathcal{F}^*[f] + b\mathcal{F}^*[g].
$$

2. Suppose $t^nf(t)\in L^1[-\infty,\infty]$ for some positive integer $n.$ Then

$$
\mathcal{F}[t^n f(t)](\lambda) = i^n \frac{d^n}{d\lambda^n} \{ \mathcal{F}[f](\lambda) \}.
$$

3. Suppose $\lambda^n f(\lambda) \in L^1[-\infty, \infty]$ for some positive integer n. Then

$$
\mathcal{F}^*[\lambda^n f(\lambda)](t) = i^n \frac{d^n}{dt^n} \{\mathcal{F}^*[f](t)\}.
$$

4. Suppose the *n*th derivative $f^{(n)}(t) \in L^1[-\infty, \infty]$ and piecewise continuous for some positive integer n , and f and the lower derivatives are all continuous in $(-\infty, \infty)$. Then

$$
\mathcal{F}[f^{(n)}](\lambda) = (i\lambda)^n \mathcal{F}[f](\lambda).
$$

5. Suppose *n*th derivative $f^{(n)}(\lambda) \in L^1[-\infty, \infty]$ for some positive integer n and piecewise continuous for some positive integer n , and f and the lower derivatives are all continuous in $(-\infty, \infty)$. Then

$$
\mathcal{F}^*[f^{(n)}](t) = (-it)^n \mathcal{F}^*[f](t).
$$

6. The Fourier transform of a translation by real number a is given by

$$
\mathcal{F}[f(t-a)](\lambda) = e^{-i\lambda a} \mathcal{F}[f](\lambda).
$$

7. The Fourier transform of a scaling by positive number b is given by

$$
\mathcal{F}[f(bt)](\lambda) = \frac{1}{b}\mathcal{F}[f](\frac{\lambda}{b}).
$$

8. The Fourier transform of a translated and scaled function is given by

$$
\mathcal{F}[f(bt-a)](\lambda) = \frac{1}{b}e^{-i\lambda a/b}\mathcal{F}[f](\frac{\lambda}{b}).
$$

Examples

 $\bullet~$ We want to compute the Fourier transform of the rectangular box function with support on $[c, d]$:

$$
R(t) = \left\{ \begin{array}{ll} 1 & \text{if } c < t < d \\ \frac{1}{2} & \text{if } t = c, d \\ 0 & \text{otherwise.} \end{array} \right.
$$

Recall that the box function

$$
\Pi(t) = \begin{cases} 1 & \text{if } -\pi < t < \pi \\ \frac{1}{2} & \text{if } t = \pm \pi \\ 0 & \text{otherwise.} \end{cases}
$$

has the Fourier transform $\hat{\Pi}(\lambda) = 2\pi$ sinc λ , but we can obtain R from Π by first translating $t \to s = t - \frac{(c+d)}{2}$ and then rescaling $s \to \frac{2\pi}{d-c} s$:

$$
R(t) = \Pi(\frac{2\pi}{d-c}t - \pi\frac{c+d}{d-c}).
$$

$$
\hat{R}(\lambda) = \frac{4\pi^2}{d-c}e^{i\pi\lambda(c+d)/(d-c)}\text{sinc}(\frac{2\pi\lambda}{d-c}).
$$
 (1.2.9)

Furthermore, from (??) we can check that the inverse Fourier transform of \hat{R} is R , i.e., $\mathcal{F}^*(\mathcal{F})R(t) = R(t)$.

 $\bullet\,$ Consider the truncated sine wave

$$
f(t) = \begin{cases} \sin 3t & \text{if } -\pi \le t \le \pi \\ 0 & \text{otherwise} \end{cases}
$$

with

$$
\hat{f}(\lambda) = \frac{-6i\sin(\lambda\pi)}{9 - \lambda^2}.
$$

Note that the derivative f' of $f(t)$ is just $3g(t)$ (except at 2 points) where $g(t)$ is the truncated cosine wave

$$
g(t) = \begin{cases} \cos 3t & \text{if } -\pi < t < \pi \\ -\frac{1}{2} & \text{if } t = \pm \pi \\ 0 & \text{otherwise.} \end{cases}
$$

We have computed

$$
\hat{g}(\lambda) = \frac{2\lambda \sin(\lambda \pi)}{9 - \lambda^2}.
$$

so $3\hat{g}(\lambda) = (i\lambda)\hat{f}(\lambda)$, as predicted.

• Reversing the example above we can differentiate the truncated cosine wave to get the truncated sine wave. The prediction for the Fourier transform doesn't work! Why not?

$8\,$

$1.2.2$ Fourier transform of a convolution

The following property of the Fourier transform is of particular importance in signal processing. Suppose f, g belong to $L^1[-\infty, \infty]$.

Definition 2 The convolution of f and g is the function $f * g$ defined by

$$
(f * g)(t) = \int_{-\infty}^{\infty} f(t - x)g(x)dx.
$$

Note also that $(f*g)(t)=\int_{-\infty}^{\infty}f(x)g(t-x)dx,$ as can be shown by a change of variable.

Lemma 1 $f * g \in L^1[-\infty, \infty]$ and

$$
\int_{-\infty}^{\infty} |f * g(t)| dt = \int_{-\infty}^{\infty} |f(x)| dx \int_{-\infty}^{\infty} |g(t)| dt.
$$

Sketch of proof:

$$
\int_{-\infty}^{\infty} |f * g(t)| dt = \int_{-\infty}^{\infty} \left(\int_{-\infty}^{\infty} |f(x)g(t-x)| dx \right) dt
$$

$$
= \int_{-\infty}^{\infty} \left(\int_{-\infty}^{\infty} |g(t-x)| dt \right) |f(x)| dx = \int_{-\infty}^{\infty} |g(t)| dt \int_{-\infty}^{\infty} |f(x)| dx.
$$

Theorem 4 Let $h = f * g$. Then

$$
\hat{h}(\lambda) = \hat{f}(\lambda)\hat{g}(\lambda).
$$

Sketch of proof:

$$
\hat{h}(\lambda) = \int_{-\infty}^{\infty} f * g(t)e^{-i\lambda t}dt = \int_{-\infty}^{\infty} \left(\int_{-\infty}^{\infty} f(x)g(t-x)dx\right)e^{-i\lambda t}dt
$$

$$
= \int_{-\infty}^{\infty} f(x)e^{-i\lambda x} \left(\int_{-\infty}^{\infty} g(t-x)e^{-i\lambda (t-x)}dt\right)dx = \int_{-\infty}^{\infty} f(x)e^{-i\lambda x}dx \ \hat{g}(\lambda)
$$

$$
= \hat{f}(\lambda)\hat{g}(\lambda).
$$

 \Box

$$
\color{red}9
$$

Exercise 9 Let $f(t)$ be defined for all $t > 0$ and extend it to an odd function on the real line, defined by

$$
G(t) = \begin{cases} f(t) & \text{if } t > 0, \\ -f(-t) & \text{if } t < 0. \end{cases}
$$

By applying the results of Exercise 7 show that, formally,

$$
f(t) = \frac{2}{\pi} \int_0^\infty \sin \alpha t \, d\alpha \int_0^\infty f(s) \sin \alpha s \, ds, \quad t > 0.
$$
 (1.4.15)

Find conditions on $f(t)$ such that this pointwise expansion is rigorously correct.

1.5 Relations between Fourier series and Fourier integrals: sampling

For the purposes of Fourier analysis we have been considering signals $f(t)$ as arbitrary $L^2[-\infty, \infty]$ functions. In the practice of signal processing, however, one can treat only a finite amount of data. Typically the signal is digitally sampled at regular or irregular discrete time intervals. Then the processed sample alone is used to reconstruct the signal. If the sample isn't altered. then the signal should be recovered exactly. How is this possible? How can one reconstruct a function $f(t)$ exactly from discrete samples? The answer, Of course, this is not possible for arbitrary functions $f(t)$. The task isn't hopeless, however, because the signals employed in signal processing, such as voice or images, are not arbitrary. The human voice for example is easily distinguished from static or random noise. One distinguishing characteristic is that the frequencies of sound in the human voice are mostly in a narrow frequency band. In fact, any signal that we can acquire and process with real hardware must be restricted to some finite frequency band. In this section we will explore Shannon-Whittaker sampling, one way that the special class of signals restricted in frequency can be sampled and then reproduced exactly. This method is of immense practical importance as it is employed routinely in telephone, radio and TV transmissions, radar, etc. In later chapters we will study other special structural properties of signal classes, such as sparsity, that can be used to facilitate their processing and efficient reconstruction.

Definition 3 A function f is said to be frequency band-limited if there exists a constant $\Omega > 0$ such that $\hat{f}(\lambda) = 0$ for $|\lambda| > \Omega$. The frequency $\nu = \frac{\Omega}{2\pi}$ is called the Nyquist frequency and 2ν is the Nyquist rate.

 20

1.6 Relations between Fourier series and Fourier integrals: aliasing

Another way to compare the Fourier transform with Fourier series is to periodize a function. The periodization of a function $f(t)$ on the real line is the function

$$
P[f](t) = \sum_{m = -\infty}^{\infty} f(t + 2\pi m)
$$
 (1.6.18)

Then it is easy to see that $P[f]$ is 2π -periodic: $P[f](t) = P[f](t + 2\pi)$, assuming that the series converges. However, this series will not converge in general, so we need to restrict ourselves to functions that decay suffiuciently rapidly at infinity. We could consider functions with compact support, say infinitely differentiable. Another useful but larger space of functions is the Schwartz class. We say that $f \in L^2[-\infty, \infty]$ belongs to the *Schwartz class* if f is infinitely differentiable everywhere, and there exist constants $C_{n,q}$ (depending on f) such that $|t^n \frac{d^q}{dt^q} f| \leq C_{n,q}$ on R for each $n, q = 0, 1, 2, \cdots$. Then
the projection operator P maps an f in the Schwartz class to a continuous function in $L^2[0, 2\pi]$ with period 2π . (However, periodization can be applied to a much larger class of functions, e.g. functions on $L^2[-\infty,\infty]$ that decay as $\frac{c}{l^2}$ as $|t| \to \infty$.) Assume that f is chosen appropriately so that its periodization is a continuous function. Thus we can expand $P[f](t)$ in a Fourier series to obtain

$$
P[f](t) = \sum_{n=-\infty}^{\infty} c_n e^{int}
$$

where

$$
c_n = \frac{1}{2\pi} \int_0^{2\pi} P[f](t) e^{-int} dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} f(t) e^{-int} dt = \frac{1}{2\pi} \hat{f}(n)
$$

where $\hat{f}(\lambda)$ is the Fourier transform of $f(t)$. Then,

$$
\sum_{n=-\infty}^{\infty} f(t + 2\pi n) = \frac{1}{2\pi} \sum_{n=-\infty}^{\infty} \hat{f}(n)e^{int},
$$
 (1.6.19)

and we see that $P[f](t)$ tells us the value of \hat{f} at the integer points $\lambda = n$. but not in general at the non-integer points. (For $t = 0$, equation (1.6.19) is known as the Poisson summation formula. If we think of f as a signal, we see that periodization $(1.6.18)$ of f results in a loss of information. However, if f vanishes outside of $[0, 2\pi)$ then $P[f](t) \equiv f(t)$ for $0 \le t < 2\pi$ and

$$
f(t) = \sum_{n} \hat{f}(n)e^{int}, \quad 0 \le t < 2\pi
$$

24

UNIT WISE OBJECTIVE BITS:

FOURIER SERIES AND POLYPHASE CIRCUITS.

A) STATE WHETHER THE GIVEN STATEMENT IS TRUE OR FALSE.

- 1) Fourier series is a trignometric series.
- 2) All fourier series are convergent.
- 3) All functions of time can be transformed into equivalent fourier series.
- 4) Perodic functions of period T is having the property $f(t) = f(t + nT)$, where n is any integer.
- 5) All odd functions of periodic nature turns into a fourier sine series .
- 6) If a function is even then the fourier series only contains cosine series.

7) In order to represent a periodic function into an equivalent fourier series the function must satisfy Drichlets conditions for convergence.

- 8) First harmonics are called as fundamental harmonics.
- 9) Active power for polyphase circuit is sum all respective phase powers.
- 10) There are only two possible phase sequence for 3 phase circuit.

B) FILL IN THE BLANKS.

- 1) For a sinusoids average value is given by ________ in terms of maximum value.
- 2) Form factor is the ratio of
- 3) Ratio of maximum value of sinusoids to rms value is factor.
- 4) J is complex operator defined by
- 5) For a 3-phase balanced star system, line voltages are __ deg. Lead of their resp. phase voltages.
- 6) The following property is true for _______ function, $f(t) = f(-t)$.
-
- 8) The fourier series coefficient of c f(t) are c times the corresponding fourier coefficient of
- 9) The fourier series of periodic function f (t) of period T is given by \sim $\sqrt{2}$
- 10) Fundamental term of fourier series in above series given by _____.

C) MULTIPLE CHOICE OBJECTIVE QUESTIONS.

- 1) Drichlets condition for fourier series
	- a) finite no. of discontinuties in $f(t)$
	- b) all discontinuties bounded in f(t)
	- c) finite no. of maxima and minima in $f(t)$
	- d) all of the above
- 2) Half wave symmetry is expressed by
	- a) $f(t) = -f(t + T/2)$
	- b) $f(t) = f(t + T/2)$
	- c) $f(t) = -f(-t)$
- d) none of the above
- 3) Half wave symmetry is also called as
	- a) even symmetry
	- b) odd symmetry
	- c) rotational symmetry
	- d) none of the above.

4) If $f(t)$ and $g(t)$ have period T and a, b are constants then function

- $P(t) = a f(t) + b g(t)$ will have period
	- $a)$ T
	- $b) 2T$
	- $c) T/2$
	- d) None of the above.
- 5) A periodic function primitive period is f(t)equals to
	- a) constant
		- b) zero
		- $c)$ t
- d) infinity.
- 6) A plot showing each of the harmonics amplitudes in the wave is called
	- a) discrete spectra
		- continucous spectra b)
		- phase spectra $c)$
		- \overline{d} none of the above
- 7) Convergence of fourier series will be faster in case of
	- a) sine wave
	- b) rectangular wave
	- c) ramp wave
	- d) impulse wave
- 8) The order in which the emfs of phase attain their maximum value is called
	- a) phase sequence
	- b) phase
	- c)harmonics
	- d)none of the above.

9)In a 3-phase , 3 -wire system if a unbalanced load is present then for analysis following theorem is needed

- a) Millmans theorem
- b) Superposition theorem
- c) Nortans theorem
-
- d) None of the above
- 10)For a star connected 3-phase, 3-wire system , the neutral current is zero, if
- a) load is balanced
- b) supply is balanced
- c) load and supply both balanced
- d) always zero.

D) ANSWER THE FOLLOWING QUESTIONS IN ONE SENTENCE

- 1) What is a periodic function?
- 2) What is odd function ?
- 3) What is even function ?
- 4) What is rotational symmetry ?
- 5) How average value is found from fourier series ?
- 6) Express fourier series in exponential form.
- 7) What is fundamental harmonics term in fourier series ?
- 8) What is phase sequence ?
- 9) What is use of millmans theorem in polyphase circuit ?
- 10) How powers are calculated in 3 phase system?

E) ANSWER THE FOLLOWING QUESTIONS IN BRIEF

- 1) Explain Fourier series.
- 2) Explain odd symmetry and its significance.
- 3) Explain even symmetry and its significance.
- 4) Explain rotational symmetry and its significance.
- 5) Explain how fourier coefficients are calculated.
- 6) Explain the exponential form of fourier series and relation between its coefficients to original fourier series coefficients.
- 7) Draw 3-phase star system and give all the rotations.
- 8) Prove that for 3-phase star system line voltages are 30 deg ahead of their resp. phase voltages.
- 9) Draw phasor dia. of 3- phase star system.
- 10) Explain star delta load transformation.

F) SOLVE THE FOLLOWING PROBLEMS

- 1) In 3- phase, 400 V system Calculate average value of phase voltage, maximum value of phase voltage and line voltage.
- 2) In 3- phase, 400 V delta system ,repeat above problem.
- 3) Transform $(3+j4)$ into its equivalent polar form.
- 4) Transform 41_30 into its equivalent rectangular form.
- 5) If $v = 3001 30$ and $I = 31 45$ in a system calculate the complex power S.
- 6) For the above problem . calculate active and reactive power .
- 7) Determine the symmetry present in the following functions and comment on fourier series. $a)$ sin t b) cos t
- 8) Determine the symmetry present in the following functions and comment on fourier series. b) $f(wt) = A$ $0 < wt < PI$ $a)$ t
	- $f(wt) = -A$ PI <wt < 2.PI
- 9) Calculate the average value of following functions. $a)$ sin t $b)$ t
- 10) Calculate the average value of following functions. b) Sq. t a) cos t
- 11) Determine the Fourier series of the wave shown in fig.

- 12) A 3-phase, 50 Hz star supply is supplying a combination of star and delta loads, find currents, loads are balanced,
	- impedance per branch for star load \therefore 2 | 30 impedance per branch for delta load : 21 - 30.

13) 3- phase ,3-wire connected 400 V supply is supplying following loads ,calculate line currents impedance of each line from generator terminal to load terminal is $z = 1 + j4$ ohm., $Zry = 30 + 40$ ohm, $Zbr = 100$ ohm, $Zyb = 60 - 130$ ohm.

14)A balanced star conneced load of 150 Kw takes a leading current of 100 A with a line voltage of 11 Kv, 50 Hz. Determine the per phase value of the elements.

15)A star connected load consisting of a pure inductance and two resistors , is connected to a symmetrical 3- phase supply. If the numerical value of all the branch impedance are the same, determine the voltage across each branch as a% of line voltage.

16)A symmetrical 3- phase ,440 V system supplies a star connected load, the branch impedances are

 $Z1 = 101$ 30 ohm $Z2 = 121$ 45 ohm $Z3 = 151$ 40 ohm.

Assuming neutral of the supply to be earthed, calculate voltage to earth of star point, phase sequence is R-Y-B.

17)On a symmetrical 3-phase system, phase sequence R-Y-B, a capacitive reactance of 8 ohm is connected across YB and a coil R+jX is connected across RY . determine R and X , the line current Iy=0.

3 phase circuit:

Q.1) A three phase , 50 Hz star - connected balanced source has a per phase voltage of 231 volts. The 3- phase load is as follows: Between R ph. & Load neutral: 100 ohms resistor. Between Y ph. & Load neutral: 100 ohms inductive reactance. Between B ph. & Load neutral: Open Circuit. Find the voltage between the source neutral and the load neutral, and draw the phasor diagram.

Fourier Transform:-

Q.1) Explain the fourier transform of a single pulse of duration T sec and height V. Discuss its amplitude spectrum and phase spectrum.

Q.2) A voltage from a source varies as :

 $V(\Theta) = 100$ volts, from $\Theta = 0$ to $\Theta = \pi$

 $V(\Theta) =$ Zero volts, from $\Theta = \pi$ to $\Theta = 2\pi$

Find its average value and the 3rd harmonic component. Assume the fundamental frequency as 50 Hz.

Q.3) If the voltage in Q.2 above is fed to an R-L-C series circuit consisting of R= 20 ohms, $L = 0.5$ H, $C = 2.254$ micro-Farads, find the D.C component of current and the third harmonic current.

LAPLACE TRANSFORM

A) STATE WHETHER THE GIVEN STATEMENT IS TRUE OR FALSE.

1) In a system of constant inductance current can change instantaneously.

- 2) In a system of constant capacitance voltage can change instantaneously.
- 3) Inductor when ckt. is allowed to relax for infinite time will behave as open circuit.
- 4) Capacitor when ckt. is allowed to relax for infinite time will behave as short circuit.
- 5) Laplace transform method was invented by heavyside.

6) If the switching is done in n/w. consisting of resistors, resistor will behave differently for transient time.

- 7) If an impulse of a current is given , the capacitor voltage can change instantaneously.
- 8) If an impulse of voltage is given, the inductor current can change instantaneously.
- 9) Time constant for RL series circuit is L/R.
- 10) Time constant for RC series circuit is RC.

B) FILL IN THE BLANKS.

1) Initial condition for unchaged capacitor is

-
- 3) First shifting property of laplace trnsform is ______.
- 4) Laplace transform of unit step function is _
- 5) Ramp function can be obtained from unit step function by the process of _______.
- 6) Impulse function is also known as
- 7) Laplace transform of delayed unit step function, by a , is
- 6) If $g(t)$ and $f(t)$ are functions of time and $G(s)$ and $F(s)$ are thir laplace transform resp., then inverse laplace transform of F(s) G(s) is given by _______ theorem.
- 7) Laplace transform method can be used for solving differentential equations.

C) MULTIPLE CHOICE OBJECTIVE QUESTIONS.

- 1) Laplace transform of unit step function is
	- a) $1/s^2$
	- $b)$ $1/s$
- $c) s$
- $d) s²$
- 2) Laplace transform of ramp function is
	- a) $1/s^2$
	- $b)$ $1/s$
	- $c) s$
	- $d) s²$

3) Inverse laplace trnsform of 1 is _____ function.

- 4)If F(s) is laplace transform of f(t) then LT of $e^{\alpha t}$ f(t) is
	- $a)$ $F(s-a)$
	- b) $F(s+a)$
	- c) $e^{3t}F(s)$
	- d) none of the above.

5)Laplace transform of $e^{\pm i\epsilon}$

- a) $1/(s+a)$
- b) $1/(s a)$
- $c)$ a/s
-
- $d)$ s/a.
- 6) Convolution theorem is used to find inverse laplace transform of
- a) product of two transform
- b) quotient
- c) addition
- d) none of the above
- 7)If $F(s)$ is laplace transform of $f(t)$, then laplace transform of a $f(t)$, where a is constant, is
	- $a)$ $aF(s)$
	- b) $sF(a)$
	- c) $F(as)$
	- $d)$ $F(sa)$
- 8) Final condition for inductor with current is
	- a) current source
	- b) short ckt.
	- c) current source with short ckt. in series
	- d) current source with short ckt. in parallel.

D) ANSWER THE FOLLOWING QUESTIONS IN ONE SENTENCE.

- 1) Who invented the laplace transform method ?
- 2) What are the properties of laplace transforms?
- 3) What is first shifting priperty?
- 4) What is second shifting property?
- 5) What is the relation between ramp function and parabolic function?
- 6) What is convolution theorem?
- 7) What are different methods to find out inverse laplace transform ?
- 8) What is the waveform synthesis?
- 9) What are the initial conditions ?
- 10) What are the final conditions?

E) ANSWER THE FOLLOWING QUESTIONS IN BRIEF

- 1) Prove convolution theorem.
- 2) Write in brief about partial fraction method.
- 3) Prove first shifting property.
- 4) What are the advantages of laplace transform?
- 5) Write down the steps for solving network with laplace transform method.
- 6) Discuss all unit functions and their properrties.
- 7) State initial conditions and prove them.
- 8) State final conditions and prove thm also state where they are applicable.
- 9) Discuss behaviour of RL series circuit.
- 10) Discuss behaviour of RC series circuit with switch operated $at t = 0$.

F) SOLVE THE FOLLOWING PROBLEMS

- 1) Find out laplace transform of the folowing.
- $c) cos (at)$ $a) t$ b) $sin(at)$
- 2) Find out inverse laplace transform of following.
	- a) $1/(s^2+ w^2)$
	- $b) 1$
	- $c)$ 1/s.
- 3) Find out inverse laplace transform of $(2s + 3) / (s² + 3s + 2)$ by partial fraction method.
- 4) If $f(t) = \sin t$ and is perodic function. Find out its laplace transform.

5) Find out the initial and final value of $(s+b)/s(s+a)$.

6) Find the following function as combination of step $\&$ ramp function and obtain Laplace transform

NETWORK FUNCTIONS

A) MULTIPLE CHOICE OBJECTIVE QUESTIONS.

- 1) The driving point impedance is defined as
- 2) The Transfer impedance is defined as the ratio of transform voltage at one port to transform current at
- the function is said to have simple poles and zeros only if $3)$
	- a) the poles are not repeated
	- b) the zeros are not repeated
	- c) both poles and zeros are not repeated
	- d) none of the above
- 4) The necessary condition for driving point function is
	- a) The real part of all the poles and zeros must not be zero or negligible
	- b) The polynomial P(s) and Q(s) may not have any missing terms between the highest and lowest degree unless all even or odd terms are missing.
	- c) The degree of P(s) and Q(s) may differ by more than one
	- d) The lowest degree in $P(s)$ and $Q(s)$ may differ by more than two
- 5) The necessary condition for transfer function is that
	- a) The coefficient in polynomial P(S) and Q(s) must be real
	- b) Coefficient in Q(s) may be negligible
	- c) complex and imaginary poles and zeros may not conjugate
	- d) if the real part of pole is zero then that pole must be multiple
- 6) The system is said to be stable, if and only if
	- a) all poles lie on right half of s plane
	- b) some poles lie on right half of s plane
	- c) all poles does not lie on right half of s plane
	- d) none of the above
- 7) The transfer voltage gain is defined as
	- a) The ratio of transform voltage at one port to current transform at other port.
	- b) The ratio of transform voltage at one port to voltage transform at other port.
	- c) Both a) and b) d) none of the above.
- 8) The transform current gain is defined as
	- a) The ratio of transform current at one port to current transform at other port.
	- b) The ratio of transform voltage at one port to voltage transform at other port.
	- c) The ratio of transform current at one port to voltage transform at other port.
	- d) None of the above.
- 9) The driving point admittance is defined as
	- a) The ratio of transform voltage at one port to current transform at other port.
	- b) The ratio of transform current to voltage transform at same port.
	- c) The ratio of transform current at one port to voltage transform at other port.
	- d) none of the above.
- 10) Transfer admittance is defined as
- a) The ratio of transform voltage at one port to current transform at other port.
- b) The ratio of transform current to voltage transform at same port.
- c) The ratio of transform current at one port to voltage transform at other port.
- d) none of the above.

B) FILL IN THE BLANKS.

- 1) The pair of terminals is customarily connected to the energy source which is driving force of the network so that pair of terminal is known as _______ of network.
- 2) Because of the similarity of impedance and admittance the two quantities are assigned one name as
- 3) It is conventional to define _______ as the ratio of an output quantity to an input quantity.
- 4) When r poles or zeros have the same value , the pole or zero is said to be of

5) When the variable s has the value such the network function vanishes , that complex frequencies are known as ___________of network function.

6) When the variable s has values such that the network function becomes infinite , that complex frequencies are known as ______ of network function.

7) If the pole or zero is not repeated it is said to be

For any _____ function , the total no. of poles is equal to total no. of zeros.

8) A one terminal pair network is an open circuit for pole frquencies and for zero frequencies.

9) Neywork function with _____ in right half of s plane are known as non minimum phase.

C) STATE WHETHER THE GIVEN STATEMENT IS TRUE OR FALSE

- 1) A function relating currents or voltages at different parts of network, called transfer function.
- 2) The driving point impedance function is defined as the ratio of current transform to voltage transform at same port.
- 3) Network function having n no. of zeros and m no. of poles and if $n>m$, then poles at infinity is of degree n - m.
- 4) Network function having n no. of zeros and m no. of poles and if $n < m$, then poles at infinity is of degree m-n.
- 5) The pole represents a frequency at which network function blows up.

6) Necessary condition for driving point as well as transfer function, poles and zeros must be conjugate if imaginary or complex

7) Network function with left half plane zeros are classified as non minimum phase.

8) Active network is stable if transfer function relating output to input has poles which are confined to left half of s plane.

9) An equivalent requirement for a stable system is that bonded input must give rise to bounded output.

10) Contours of constant wn are straight lines parallel to jw axis of s plane.

D) ANSWER THE FOLLOWING QUESTIONS IN ONE SENTENCE

- 1) What is the network function?
- 2) Define driving point and transfer function.
- 3) Define driving point impedance and transfer function.
- 4) Define transfer impedance and admittance function.
- 5) What is the voltage transfer and current transfer gain?
- 6) What are the poles and zeros?
- 7) What are the requirement for the stable active network?
- 8) Define minimum phase and non minimum phase function.
- 9) What is the scale factor of network function?

E) ANSWER THE FOLLOWING OUESTIONS IN BRIEF.

1) write short notes on driving point impedance and transfer function.

2) What information do poles and zeros provide in respect of network to which they relate ?

3) Breifly discuss the restriction on pole zero location in the s plane for driving point impedance function.

4) Show that first singularity of an RC admittance function is zero at origin of s plane.

5) State under what condition the concept of driving point transfer function can be used.

6) Define precisely the various transfer function of two terminal pair passive network.

7) Enumerate the important properties of driving point impedance function of one port passive network.

8) Explain how time domain response of a system can be determined from s plane plot of poles and zeros of its network function.and transform of network sources.

9) Briefly discuss the restriction on s plane zeros location in the s plane for transfer impedance function.

10) Describe the graphical procedure for finding time domain behaviour from pole zero plot.

F) SOLVE THE FOLLOWING PROBLEMS.

1) For the network shown find G12(S). Write the results in the form of polynomials in S to decide poles and zeros.

2) Find the transfer functions Z12(S) and G12(S) for the network shown in fig.

TWO PORT NETWORK

A) MULTIPLE CHOICE OBJECTIVE TYPE QUESTIONS

1) For a two port network, the o/p short circuit current was measured with a 1 V source at the i/p terminal, the value of the current gives

 $a)$ h12

 $b)$ $v12$

 $c)$ h21

 $d)$ $v21$

2)If a passive reciprocal two port network with open circuit impedance matrix Zoc is terminated in Zl ohm , the driving port impedance of overall network is ______.

3) The resistance Rab of the circuit is

a) 12 ohms

b) 10.8ohms

c) 6.75 ohms

d) 0.9 ohms

4) Determine Z parameter of T network

a) 5,8,12,0

b) 13,8,8,20

c) $8,20,13,12$

d) $5,8,8,12$

5) Find the Z parameter of the T network given by

ZA= 51_0 ; ZB = 101_90 ; ZC = 151_90

a) 51 0; 151 90; 151 90; 101 90

b) 15| 0; 10| -90; 10| -90; 25| 0

c) 15.81 | 71.57; 15| 90; 15| 90; 5| 90

d) 51 90; 151 90; 151 90; 15.81 1 71.57

6)The condition $AD-BC = 1$ for a two port network implies that the network is

a) Reciprocal network

b) Lumped element network

c) Loss less network

d) Unilateral element network

7) Two port network are connected in cascade, the combination is to be represented as a single

two port network, the parameters of the network are obtained by multiplying the individuals

a) Z parameter matrix

b) H parameter matrix

c) Y parameter matrix

d) Transmission parameter matrix

8) For two port network to be reciprocal

a) $Z11 = Z22$

b) $Y21 = Y12$

c) $H21 = -h12$

 $d)$ AD - BC = 0

B) FILL IN THE BLANKS

1) In terms of Y parameters, the H parameters are 2) In terms of transmission parameters, the z parameters are _____. 3)The ABCD parameters of the T network is 4)In terms of Z parameters, the Y parameters are 5) The Y parameters of the lattice are ______.

6)If a two port n/w is passive, then we have, with the usual notations relationship

- a) $h12 = h21$
- b) $h12 = -h21$
- c) $h11 = h22$
- d) $h11h12 h12h21 = 1$

C) STATE WHETHER THE GIVEN STATEMENT IS TRUE OR FALSE.

- 1) In ABCD parameters, o/p current flows out into the transmission line.
- 2) For a symmetrical n/w Za = Zb, where, Z11= Za + Zc, Z22= Zb + Zc
- 3) Any 4 terminal black box is represented by Z, Y and h parameters.
- 4) For the cascade connection of two networks ABCD parameters have to be multiplied.
- 5) For a series parallel connection of two networks Z parameters have to be added.
- 6) The condition for a network to be loss less in terms of ABCD parameters is A and D real and B.C imaginary.
- 7) Ya, Yb, Yc are the admittance of sub network of PI network, the short ckt. Admittance parameters Y11 will be Yb + Yc
- 8) The short ckt, admittance parameter Y22 of the above n/w is Yb + Yc.

D) SUBJECTIVE TYPE QUESTIONS

- 1) What is two port network?
- 2) What are the Z parameters? Why they are called open circuit impedance parameters? explain.
- 3) Derive the condition for the reciprocal network in terms of Z parameters.
- 4) What is the condition for the symmetrical network?
- 5) Derive the condition for the reciprocal network in terms of Y parameters.
- 6) Define open circuit admittance parameter.
- 7) Prove that $Y11 = Y22$.
- 8) What are the ABCD parameters ? Define A,B, C,D individually.
- 9) Derive $AD BC = 1$
- 10) Derive $A = D$
- 11) Express ABCD parameters in terms of Z parameters.
- 12) What are the ABCD parameters?
- 13) Derive the condition for symmetrical n/w in terms of h parameters.
- 14) Define inverse hybrid parameters and prove that $g11g12 g12g21 = 1$.
- 15) What are the various types of interconnections possible in 2 port network?

E) SOLVE THE FOLLOWING PROBLEMS

1) The n/w shown below, find Z parameters.

2) Find Y parameters for the network shown below

4) Current I1 and I2 entering at ports 1 and 2 resp. are given by following eqn.

 $II = 0.5$ VI -0.2 V2

 $12 = -0.2 \text{ V1} + \text{V2}$

Where V1 and V2 are the voltages at two ports, find Z parameters and Verify that AD-BC =1 5)In a two port network, $Z_{11} = 2$ ohm, $Z_{12} = Z_{21} = 5$ ohm, $Z_{22} = 1$ ohm,

Find (i) Y-parameters (ii) h-parameters (iii) ABCD parameters

FILTERS:

Q.1) Explain the classifications of filters in brief.

Q.2) Explain the band pass & band stop (band reject) filters.

O.3) Discuss the design procedure for the design of constant K- band pass filter in terms of nominal characteristics impedance & cut off frequencies.

Q.4) For the constant K - band pass filter, show that the resonant frequency frequency of individual arm should be the geometric mean of its two cut - off frequencies.

Q.5) Design a prototype band pass filter having the cut off freq. of 2000 Hz & 5000Hz & nominal characteristics impedance of 600 ohms.

Q.6) A n - section filter comprises a series arm inductance of 20mH & two shunt capacitors each of 0.16 micro farad. Calculate the cut off freq. & attenuation at 15 KHz. What is the value of nominal terminating impedance in band pass filter?

RESONANCE:

Q.1) Explain the meaning of the half power frequencies and derive their expressions for a series RLC circuit.

[1] The period of the signal $x(t) = 8 \sin (0.8 \pi t + \pi / 4)$ is [GATE 2010]

A. 0.4πs

B. 0.8πs

C. 1.25s

D. 2.5s

Ans:D

Answer

[2] The switch in the circuit has been closed for a long time. It is opened at $t=0$. At $t=0^+$, the current through the 1μF capacitor is [GATE 2010]

Ans: B

Answer

[3] The second harmonic component of the periodic waveform given in the figure has an amplitude of [GATE 2010]

Ans:A Answer

[4] As shown in the figure, a 1 resistance is connected across a source that has a load line v+i=100. [The current through the](http://3.bp.blogspot.com/-o3IT4NzdzaI/UK1bOEeOSOI/AAAAAAAABy0/fcbSWEo94cc/s1600/7.png) resistance is [GATE 2010]

A. 25A

B. 50A

C. 100A

D. 200A

Ans: B

Answer

[5] If the electrical circuit of figure (b) is an eqiuvalent of the coupled tank system of figure (a), then [GATE 2010]

A. A,B are resistances and C,D capacitances

B. A,C are resistances and B,D capacitances

C. A,B are capacitances and C,D resistances

D. A,C are capacitances are and B,D resistances

Ans:D

Answer

[6] If the 12Ω resistor draws a current of 1A as shown in the figure, the value of resistance R is [GATE 2010]

A. 4Ω

B. 6Ω

C. 8Ω

D. 18Ω

Ans: B

Answer

[7] The two-port network P shown in the figure has ports 1 and 2, denoted by terminals (a,b) and (c,d), respectively. It has an impedence matrix Z with parameters denoted by Z_{ij} . A 1 Ω resistor is connected in series with the network at port 1 as shown in the figure. The impedance matrix of the modified two-port network (shown as a dashed box) is [GATE 2010]

$$
\begin{pmatrix} z_{11} \!\! \!&+ 1 & z_{12} \!\! \!&+ 1\\ z_{21} \!\! \!& z_{22} \!\! \!&+ 1 \end{pmatrix}
$$

$$
\begin{pmatrix} z_{11}+1 & z_{12} \\ z_{21} & z_{22}+1 \end{pmatrix}
$$

$$
\begin{pmatrix} z_{11}+1 & z_{12} \\ z_{21}+1 & z_{22} \end{pmatrix}
$$

$$
\begin{pmatrix} z_{11} + 1 & z_{12} \ z_{21} & z_{22} \end{pmatrix}
$$

Ans:C Answer

[8] The Maxwell's bridge shown in the figure is at balance.The parameters of the inductive coil are [GATE 2010]

A. R=R2R3/R4, L=C4R2R³ B. L= R_2R_3/R_4 , R= $C_4R_2R_3$ C. $R=R_4/R_2R_3$, $L=1(C_4R_2R_3)$ D. L=R₄/R₂R₃, R=1/(C₄R₂R₃)

Ans:A

Answer

Statement for Q9 & Q10:

The L-C circuit shown in the figure has an inductance L=1mH and a capacitance C=10μF **Question [9]:** The initial current through the inductor is zero, while the initial capacitor voltage is 100V. The switch is closed at t=0. The current i through the circuit is: [GATE 2010]

- A. 5cos(5x10³t)A
- B. 5sin(10⁴t)A
- C. 10cos(5x10³t)A
- D. 10sin(10⁴t)A

Ans: D

Answer

Question [10]: The L-C circuit of statement is used to commutate a thyristor, which is initially carrying a current of 5A as shown in the figure below. The values and initial conditions of L and C are the same as in statement. The switch is closed at $t=0$. If the forward drop is negligible, the time taken for the device to turn off is [GATE 2010]

- A. 52μs
- B. 156μs
- C. 312μs
- D. 26μs

Ans: A

Answer

[11] The voltage applied to a circuit is 100√2 cos(100πt) volts and the circuit draws a current of 10√2sin(100πt +π/4) amperes. Taking the voltage as the reference phasor, the phasor representation of the current in amperes is [GATE 2011]

A. 10√2∠- π/4 B. 10∠- π/4 C. 10∠+ π/4 D. 10√2∠+ π/4 **Ans:A** Answer

University previous Question papers:

Question Bank

Network Theory Question Bank

Unit-I JNTU SYLLABUS: Three Phase Circuits

Three phase circuits: Phase sequence - Star and delta connection - Relation between line and phase voltages and currents in balanced systems - Analysis of balanced and Unbalanced 3 phase circuits - Measurement of active and reactive power. 1. (a) The power delivered to a balanced delta connected load by a 400 volt 3-phase supply is measured by two wattmeter method. If the readings of the two wattmeter are 2000 and 1500 watts respectively, calculate the magnitude of the impedance in each arm of the delta load and its resistive component? (b) A balanced delta connected load of $(2+i3)$ per phase is connected to a balanced three-phase 440V supply. The phase current is 10A. Find the i, total active power ii. Reactive power and iii. Apparent power in the circuit. [8+8] February 2008, set-1 2. (a) On a symmetrical 3-phase system, phase sequence RYB, a capacitive reactance of 8 is across YB and a coil (R+jX) cross RY. Find R and X such that $Iy = 0$. (b) Find the reading on the wattmeter when the network shown in figure 4 is connected to a symmetrical 440V, 3- φ supply. The phase sequence is RYB . $[8+8]$ February 2008, set-2 3. (a) On a symmetrical 3-phase system, phase sequence RYB, a capacitive reactance of 8 is across YB and a coil (R+jX) cross RY. Find R and X such that $Iy = 0$. (b) Find the reading on the wattmeter when the network shown in figure is connected to a symmetrical 440V, 3- φ supply. The phase sequence is RYB.[8+8] June 2009, set-1 4. (a) Derive the expressions between phase and line voltages, and phase and line Currents for balanced 3-f star connected loads, (b) A 3-phase 4 wire, 400V system feeds three loads $(10-j8)$, $(12+j0)$ and (8+j10) connected in star. Find the line currents, neutral current and total active power. [8+8] June 2009, set-2 5. What are the advantages of a poly phase system over a single phase system? [4] April/May-07 set-2, set-4 6. What is phase sequence? Explain its significance. [6] Aug/sept-07 set-1, April/May-07 set-2, set-4 7. What is the difference between RYB phase sequence and RBY phase sequence? [4] May/june-06 set-3 8. A 3-phase load has a resistance of 10Ω in each phase and is connected in i) Star ii) Delta against a 400V, 3-phase supply. Compare the power consumed in both the cases. [6] May/june-06 set-3 9. Three identical impedances of $(3+j4)$ Ω are connected in delta. Find the equivalent star network such that the line current is same when connected to the same supply? [4] Aug/sept-08 set-3, May/june-08 set-1, May/june-08 set-2 10. Derive the relationship between line and phase quantities in a 3-phase balanced, i) Star connected system and ii) Delta connected system. [8] May/june-06 set-3, Aug/sept-06 set-4 11. A balanced 3-phase mesh connected load of (8+j6) Ω per phase is connected to a 3-phase balanced, 50Hz, 230V supply. Calculate i) Line current

ii) Power factor iii) Reactive volt-ampere and iv) Total volt-ampere. [8] May/june-08 set-3. 12. A balanced 3-phase system supplied from 400V, 50Hz supply has R, L, and C are 10Ω, 1H and 100µF resp. calculate the line current, the power and power factor?[16] Aug/Sep 2008 Set 4 13. A balanced 3-phase Delta connected load absorbs a complex power of 100KVA with a lagging power factor of 0.8 when the r.m.s line to line voltage is 2400 volts. Calculate the impedance of each arm of Delta connected load. [6] Aug/Sep 2007 Set 4 14. A balanced 3-phase Delta connected load of (2+j3) Ω per phase is connected to a 3-phase balanced, 50Hz, 440V supply. Calculate i) Total active power. ii) Reactive Power and iii) Apparent power in the circuit. [8] April/May-07 set-3, set-4 15. A balanced 3-phase Delta connected load with voltage of 200V, has line currents as $I_1 = 10 \text{ L}90^\circ$, $I_2 = 10 \text{ L}150^\circ$ and $I_3 = 10 \text{ L}30^\circ$ i) What is the phase sequence? ii) What are the impedances? $[6]$ Aug/Sep 2007 Set 2 16. Three impedances of (7+j4) Ω , (3+j2) Ω , (9+j2) Ω are connected between neutral and the R, Y and B phases. The line voltage is 440V. Calculate, i) The line currents ii) The current in the neutral wire and iii) Find the power consumed in each phase and the total power drawn by the circuit. [12] Aug/Sep 2008 Set 3, May/june-08 set-1, 2 17. A symmetrical 3-phase, 3-wire, 440V Supply is connected to a star connected load. The impedances in each branch are Z₁= (2+j3) Ω , $Z_2 = (1-j2) \Omega$ and $Z_3 = (3+j4) \Omega$. Find its equivalent delta connected load. Hence, find the phase and line currents and the total power consumed in the circuits. [16] Aug/Sep 2008 Set 1, 2, and 4 18. A symmetrical 3-phase, 100V, 3-wire supply feeds an unbalanced star connected load with impedances of the load as $Z_R = 5 \text{ }\perp 0^0$ Ω , $Z_y = 2 \text{L}90^\circ \text{W}$ and $Z_8 = 4 \text{L} - 90^\circ \Omega$. Find, i) The line currents ii) Voltages across the impedances iii) The displacement neutral voltage. [10] Aug/Sep 2007 Set 2 19. Explain how power is measured in three phase delta connected load using two wattmeters. May/June-08 set-3 $[8]$ 20. Two wattmeter's are used to measure power in a 3-phase three wire load. Determine the total power, power factor and reactive power, if the two wattmeter's read i) 1000W each, both positive ii) 1000W each, but of opposite sign. [8] April/May-07 set-2, 4 21. In power measurement of 3-phase load connected by 3-phase supply by two wattmeter method, prove that tan0=-v3 (W1-W2)/ (W1+W2) for leading power factor loads. [8] May/June-06 set-1

UNIT-II JNTU SYLLABUS: D.C Transient Analysis

Transient response of R-L, R-C, R-L-C circuits (Series combinations only) for d.c- Initial conditions - Solution using differential equation approach and Laplace transform methods of solutions.

1. Derive the expression for i(t) of a R-L series circuit when DC voltage is applied to it at t=0 by closing the switch. Draw the response curve i(t) vs t, define time constant of R-L series circuit $[8]$ Nov/dec-04 set-2

2. (a) Find i(t) for $t \ge 0$, when the switch is closed at $t = 0$. The circuit was in Steady state at June 2009 set-2 $t = 0$ in the following network shown in Figure. [8]

3. (a) A dc voltage of 100V is applied in the circuit shown in figure a and the switch is kept open. The switch K is closed at $t = 0$. Find the complete expression for the current.

(b) A dc voltage of 20V is applied in a RL circuit where $R = 5$ and $L = 10H$. Find i. The time constant

ii. The maximum value of stored energy. [8+8] June 2009 set-1

4. Find \Re c (t) at t = 0 + while the switching is done from x to y at t = 0. As shown in figure. [16]

June 2009 set-2, April/May-06 set-2, May/june-06 set-1

5. Explain why the current in a pure inductance cannot change in zero time. [8]

June-05, set4

6. Explain why the voltage across a capacitor cannot change instantaneously. [2]

Nov/dec-05 set1, Nov/dec-04 set1, May/june-04 set-1

7. Derive the expression for i (t) and voltage across a capacitor Vc(t) for series R-C circuit with D.C voltage applied to it at t=0. Explain about the time constant of R-C circuit. [8] Nov/dec-04 set3

8. A DC voltage of 20V is applied in a R-L circuit where R=5 Ω and L=10H. Find the

i) Time constant

ii) The maximum value of stored energy [8]

Aug/sept-07 set-1,3,4, April/May-07 set1,3,4 Aug/sept-06 set4,1,2,3

9. A constant voltage of 100V is applied at t=0 to a series R-C circuit having R=5MΩ and C=20mF.Assuming no initial charge to the capacitor, find the expression for i, voltage across R and C.[9]

Aug/sept-06set-2

10. In the figure switch is closed at position 1 at t=0. At t=0.5msec, the switch is moved to position 2. Find the expression for the current in both the conditions and sketch the transients. [16]

May/june-06 set-4

11. What is the significance of time constant of R-L circuit? What are the different ways of defining time constant?[4] Nov/dec-05 set-1, May/june-04 set-1

12. Switch is moved from position 1 to 2 at t=0. Find the voltages $V_R(t)$ and $V_C(t)$ for t>0.[8] Nov/dec-04 set3

13. Obtain the current i(t) for t>0, using the time domain approach.[8]

Nov/dec-04 set-4, May/june-03 set-2

14. Switch is closed at t=0. Find the initial conditions at t(0⁺) for, i_1 , i_2 , V_o di₁/dt, di₂/dt, d²i₂/dt and d²i₁/dt.[16] June-05 set-4

15. Compare the classical and Laplace transform methods of solution of the network. [4]

Nov/dec-05 set-4, June-05 set-2

16. A DC voltage of 100V is applied in the circuit shown in fig. and the switch is kept open. The switch K is closed at ± 0 , find the complete expression for the current.[8]

UNIT-III

JNTU SYLLABUS: A.C Transient Analysis

Transient response of R-L, R-C, R-L-C circuits (Series combinations only) for sinusoidal excitations - Initial conditions - Solution using differential equation approach and Laplace transform methods of solutions

1 Find i (t) in the circuit for the following figure Use Laplace method. [8]

June2009 set-2

2. Derive an expression for the current response in R-L series circuit with a sinusoidal source. [16]

Aug/sept-08 set-3,4, May/june-08 set-1,3

3. In the circuit shown in fig, find the transient current when the switch is closed at t=0.Assume zero initial conditions,

4. In the circuit shown in fig, find the transient current when the switch is closed at t=0. Assume zero initial conditions. And also the initial rate of change of current.

5. A series R-L circuit with R=100 Ω , L=1H has a sinusoidal voltage source 200sin (500t+¢) applied at time when \neq =0. Find i) The expression for current

ii) At what value of ¢ must the switch be closed so that the current directly enter steady state.

6. A series RC circuit, with R=50 Ω , C=10 μ F has a sinusoidal voltage 230 $\sqrt{2}$ sin (2 Π x 50t). Find the transient response,

7. Distinguish between steady state and transient response?

8. A series R-L circuit with R=50Ω and L=0.2H, has a sinusoidal voltage source V=150sin (500t+¢)V applied at time when¢=0. Find the complete current.

9. In the two mesh network shown in fig, the switch is closed at $t=0$ and the voltage source is given by V=150 sin (1000t) V. Find the currents i1 and i2

10. An RL series circuit with R=300 Ω and L=1H has a sinusoidal applied voltage v=100cos (100t+¢) volts. If the switch is closed when $\psi = 45^\circ$, obtain the resulting current transient.

11. The RL series circuit shown in fig. is operating in the sinusoidal steady state with the switch in position 1. The switch is moved to position 2 when the voltage source is $v=100\cos 100t+45^{\circ}$) v. obtain the current transient and plot the last half cycle of steady state together with the transient to show the transition.

12. A series RLC circuit with R=5 Ω , L=0, 1H, C=500 μ F has a sinusoidal voltage v=100sin(250t+¢) volts applied at time when ϕ =0. Find the resulting current

Unit-4 JNTU SYLLABUS: Network Functions.

The concept of complex frequency, physical interpretation of complex frequency, transforms impedance and transforms circuits. Series and parallel combination of elements, terminal pairs or ports, Network functions for the one-port and two port, poles and zeros, properties of driving point functions, properties of transfer functions, necessary conditions for driving point functions, necessary conditions for transfer functions, time domain response from pole zero plot,

1. Find the driving point impedance of the network shown in fig.

- 2. Find the poles and zeros and the scale factor of the network function. $N(s) = (2s+1)/4(s2+5s+6)$. How many zeros are at infinity?
- 3. Determine the transform impedance of the network shown in fig.

4. Determine the transform impedance of the network shown in fig

5. Find the driving point impedance of the network shown in fig.

6. Determine the conditions under which the input impedance of the network shown in fig will be equal to R

7. Find the transfer function $G_{21}(s)$ and $Z_{21}(s)$ and the driving point impedance $Z_{11}(s)$ for the network shown.

8. Draw the transform network corresponding to that shown in fig. assuming all initial conditions to be zero. Also draw the simplified transform network obtained by combination of various impedances and admittances and there from obtain I(s).

A series RLC circuit has for its driving point admittance pole-zero diagrams are shown in fig. find the values of R, L, C. 9.

The Laplace transform of a voltage $v(t)$ is $V(s)=4(s+1)/(s+2)(s+3)$. Draw poles and zeros of this function and determine $v(t)$ 10. using pole-zero plot.

The transform voltage V(s) of a network is given by V(s) =4s/(s+2)(s²+2s+2) plot its pole-zero diagram and hence obtain 11. $v(t)$.

An RC circuit is shown in fig with R=3 Ω and C=1/12F. Draw the pole-zero plot for Z(S)=V(S)/I(S) 12.

UNIT-V

JNTU SYLLABUS: Network Parameters-I

Two port network parameters - Z, Y, ABCD and hybrid parameters and their relations.
1. (a) Determine the y-parameters of the network shown in figure.

(b) The Z-parameters of a two port network are $Z11=15$, $Z22=24$, $Z12=Z21=6$.

Determine

- i. ABCD parameters and
- ii. Equivalent T network. [8+8]

February2008 set-1

2. (a) Determine the ABCD parameters of the network shown in figure a.

(b) Determine the ABCD parameters of the network shown in figure. [6+10]

February2008 set-2, June-2009 set-1,April/may-07 set2,Aug/sept-06 set-1,2

3. (a) In a T network shown in figure a, $Z1 = 2 \sqcup 0^{\circ}$, $Z2 = 5 \sqcup -90^{\circ}$, $Z3 = 3 \sqcup 90^{\circ}$, find the Z-parameters.

(b) Z-parameters for a two port network are given as Z11=25, Z12=Z21=20, Z22=50. Find the equivalent T-network. [8+8] June 2009 set-2, Aug/sept-07 set-1,3, April/May-07 set-3

Determine the Z-parameters of the network shown in figure. [8] May/june-06 set1 4.

- 5. Two identical sections of the network shown in fig are connected in parallel. Obtain the Y-parameters of the resulting network and verify the result by direct calculation.[6+10] Aug/sept-08 set-1
- 6. Find the Y-parameters of the network shown in fig.[6] April/may-07 set-1,4

7. Find the Y-parameters of the network shown in fig.[16]

Aug/sept-06 set-3, May/june-06 set-4

8. Determine the ABCD parameters of the network shown in figure.[6]

April/may-07 set-2, Aug/sept-06 set-1,2

9. Find the h-parameters of the network shown in fig.[8] Aug/sept-08 set-2

10. For the h-parameters equivalent network shown in fig find the voltage gain load resistance is R_L [10] Aug/sept-07 set4, Aug/sept-06 set-4,2

- 11. The Y-parameters of the network are $Y_{11}=0.6$ mho, $Y_{22}=1.2$ mho, $Y_{12}=0.3$ mho,
	- Determine the ABCD parameters i)
	- ii) Equivalent Π network.[8] May/June-06 set-1
- 12. For the two port network shown in fig, the currents I1 and I2 are entering at port1 and port2 respectively. Find the Y,Z,ABCD parameters for the n/w. also find its equivalent Π network.[16]

Aug/sept-08 set-4

UNIT-VI NETWORK PARAMETERS-II

Cascade networks, concept of transformed network-2 port network parameters using transformed variables.

1. Find the transformed ABCD parameters of the network as shown in fig.[8] Aug/sept-08 set-2

2. Find the transformed Z-parameters for the n/w shown in fig.[16] May/june-08 set-3

- 3. Explain the concept of transformed network?
- 4. Compute the transformed Z and Y -parameters for the circuit shown in fig.[8]

Aug/sept-08 set-3

5. Find the transformed Z parameters for the circuit shown in fig.[8] June-2009 set-3

6. Find the transformed Z parameters for the circuit shown in fig.[8] June-2009 set-4

7. Find the Z-parameters in the circuit of Fig.

8. Find the Y-parameters in the circuit of Fig.

9. Two two-port networks a and b with transmission parameters Ta and Tb are connected in cascade

v.	\mathbf{T}_a	\mathbf{V}_{a} \mathbf{V}	\mathbf{V}_{1b} \mathbf{T}_{b}	\mathbf{v}_2

Given I_{2a} ¼ _I1b and V_{2a} ¼ V_{1b} , find the T-parameters of the resulting two-port network.

10. Find the T- and Z-parameters of the network in Fig. The impedances of capacitors are given. Use the cascade connection rule.

11. Find the Z-parameters of the two-port circuit of Fig.

12. Find the Z-parameters of the two-port circuit of Fig.

UNIT-VII FILTERS-I JNTU Syllabus

\Low pass, high pass, band pass, band elimination, prototype filter design

- 1. Design a low pass T and Π section filters having a design impedance $R_0 = 600\Omega$ and cut-off frequency = 2000Hz
- 2. Design a proto type section of band pass filter having cut-off frequencies of 1 KHz and, 5 KHz and a design impedance of 6000
- 3. Design a proto type section of band pass filter having cut-off frequencies of 12KHz and 16 KHz and a design impedance of 600Ω .
- 4. A constant K low pass filter is designed to cut-off at a frequency of 1000Hz and the resistance of the load circuit is 50 Ω . Calculate the values of the corresponding components required.
- 5. Design a constant k low pass filter Π and T sections at a frequency of 5KHz and a design impedance of 1000 Ω . Calculate the attenuation constant at a frequency of 6kHz and phase shift of 1kHz.
- 6. The elements of a T section of a constant K low pass filter are as shown Inductance=50m H each And Capacitance=0.01µF, calculate the cut-off frequency, pass band and the nominal impedance.
- 7. The elements of a T section of a constant K low pass filter are as shown Inductance=50m H each And Capacitance=0.01µF, calculate the cut-off frequency and characteristic impedances at a frequency of 1KHz and 5KHz. Also find the attenuation and phase shift at 1 kHz and 5 kHz.
- 8. Design a constant k-high pass filter to have a cut-off frequency of 2 KHz and a design impedance of 100Ω.
- 9. Design a band pass filter with cut off frequencies of 2000Hz and 5000Hz and a design impedance of 500Q.
- 10. Design a constant k high pass filter to cut off at 10KHz and design impedance of 600 Ω .
- 11. Design a k low pass filter to cut off at 2500Hz and design impedance of 700Q.
- 12. Design a constant k filter to eliminate band of frequencies lying between 2000Hz and 5000Hz with a design impedance of 600Ω.

UNIT-VIII

JNTU Syllabus

UNIT-VIII Fourier analysis of A.C circuits. The Fourier theorem, consideration of symmetry, exponential form of Fourier series, line spectra and phase spectra, Fourier integrals and Fourier transforms, properties of Fourier transforms

1. Find the Fourier series for the wave form shown in fig. the wave form is continuous for 0<6t<2H and given by $f(t)=(10/2\Pi)\omega t$, with discontinuous at $\omega t=n2\Pi$.

2. Find the exponential Fourier series for the waveform shown in fig. using the coefficients of this exponential series obtain a_n and b_n of the trigonometric series.

3. Find the trigonometric Fourier series for the square wave shown in fig and plot the line spectrum.

4. Find the trigonometric Fourier series for the triangular wave shown in fig. and plot the spectrum.

5. Find the trigonometric Fourier series for the saw tooth wave shown in fig. and plot the spectrum.

6. Find the trigonometric Fourier series for the wave shown in fig. and plot the spectrum?

7. Find the exponential Fourier series for the waveform shown in fig and plot the spectrum?

8. Find the exponential Fourier series for the half wave rectifier in the interval 0<6t<Π, f (t) =V sin6t. And from Π to 2Π, $f(t)=0$.

9. Find the exponential Fourier series for the waveform shown and plot the spectrum.

- 10. What are the properties of Fourier series?
- 11. A pure inductance of 0.01H has an applied current of triangular waveform shown in fig. where $\acute{\omega}$ =500rad/sec. obtain the exponential Fourier series for the current and find the series expression for the voltage across the inductance V_L.

- 12. Find the Fourier transform of the square pulse
 $X(t) = \begin{cases} 1 & \text{for } -T < t < T \end{cases}$ 0 otherwise
- 13. Find the spectrum of $e^{-at} U(t)e^{-at}u(-t)$, a > 0, shown in Fig.

Unit-wise Quiz Questions and long answer questions

TRANSIENT ANALYSIS

O1) Capacitor acts like for the a.c. signal in the steady state a) open b)closed c) not open not close Ans: (c) d'inone Q2) Double energy transient are produced in circuits consisting of a) two or more resistors b) resistance and inductance c) resistance and capacitance d) resistance, inductance and capacitance $Ans(d)$ Q3) The transient current in a loss free L-C circuit when excited from an ac source is a /an sine wave a) over damped b) under damped c) un damped d) critically damped Ans $@$ O4)The Transient current in an R-L-C circuit is oscillatory when $a)$ R=0 b) $R > 2\sqrt{L/C}$ c) $R \leq 2\sqrt{L/C}$ d) $R = 2\sqrt{L/C}$ Ans: (c) Q5) Which of the following does not have the same units as the others? The symbols have their usual meanings c) \sqrt{LC} d) $1/\sqrt{LC}$ a) L/R $b)RC$ Ans: (c,d) Q6) A DC voltage source is connected across a series RLC circuit, under steady state conditions, the applied DC voltage drops entirely across the b) L only $d)$ R & L combinations a) R only $c)$ C only Ans: (c) Q7) Consider a DC voltage source connected to a series RC circuit. When the steady state reaches, the ratio of energy stored in the capacitor to the total energy supplied by the voltage source is equal to $a) 0.362$ $b) 0.500$ $c) 0.632$ $d) 1.00$ Ans: (b) O8) For a second order system, damping ratio δ is $0 < \delta < 1$, then the roots of the characteristic polynomial are a) real but not equal b) real and equal c) complex conjugates d) imaginary Ans: © Q9) The response of an LCR circuit to a step input is If the TF has a) over damped 1) poles on -ve real axis b) critically damped 2) poles on imaginary axis c) oscillatory 3) multiple poles on +ve real axis 4) poles on +ve real axis abc 5) multiple poles on -ve real axis a) 125 b) 152 c) 345 Ans: (b) d) 154 Q10) A rectangular voltage pulse of magnitude V and duration T is applied to a series combination of R and C. The max voltage developed across the capacitor is d) Ve^{-T/RC} a) $V(1-e^{-T/RC})$ b) VT/RC Ans: (a) $c) V$ Q11) An ideal voltage source will charge an ideal capacitor a) in infinite time b) exponentially c) instantaneously d) none Ans: (c) Q12) Energy stored in a capacitor over a cycle, when excited by an a.c source is a) same as that due to a dc source of equivalent magnitude b) half of that due to a dc source of equivalent magnitude c) zero d) none Ans: © Q13) Two coils having equal resistance but different inductances are connected in series. The time constant of the series combination is the Ans: (b) a) Sum of the time constants of the individual coils b) Average of the time constants of the individual coils c) Geometric mean of the time constants of the individual coils d) Product of the time constants of the individual coils Q14) An inductor at $t=0$ with initial current I_0 acts as a) Short c) current source d) voltage source Ans: \odot b) open Q15) An inductor L carries steady state current I_0 , suddenly at time t=0 the inductor is removed from circuit and connected to a resistor R. The current through the inductor at time t is equal b) $I_0(1-e^{-Rt/L})$ d) $I_0(1-e^{+Rt/L})$ a) $I_0e^{-Rt/L}$ c) $I_0e^{+Rt/L}$ Ans: (a) Q16) Transient current in a circuit results from

a) voltage applied to the circuit b) impedance of the circuit c) changes in the stored energy in inductors and capacitors d) resistance of the circuit Ans: \circledcirc Q17) A two terminal black box contains one of the RLC elements. The black box is connected to a 220 volts ac supply. The current through the source is I. When a capacitance of 0.1 F is inserted in series between the source and the box, the current through the source is 2I. The element is Ans: (b) a) a resistance b) an inductance c) a capacitance of $0.5 F$ d) not identifiable on the basis of the given data Q18) A two terminal black box contains a single element which can be R.L.C or M. As soon as the box is connected to a dc voltage source, a finite non-zero current is observed to flow through the element. The element $is a$ /an b) inductance c) capacitance d) Mutual inductance a) Resistance Ans: (b) O19) If an RL circuit having angle φ is switched in when the applied sinusoidal voltage wave is passing through an angle θ , there will be no switching transient if a) θ - φ =0 $b) \theta + \omega = 0$ c) θ - φ =90 d) θ + φ =90 Ans: (a) O20) The correct sequence of the time constants of the circuit shown in the increasing order is L R Т. $L/2$ $2R$ R $1)$ $\left(4\right)$ $\mathcal{L}_{\mathsf{ww}}\Gamma$ L R Ľ a) $1-2-3-4$ b) $4-1-2-3$ c) $4-3-1-2$ d) $4-3-2-1$ Ans: (c) Q21) In a circuit the voltage across an element is $v(t) = 10$ (t-0.01) e^{-100t}V. The circuit is a) Un damped b) under damped c) critically damped d) Over damped $Ans:(c)$ $Q22$) A unit step voltage is applied at $t=0$ to a series RL circuit with zero initial conditions a) It is possible for the current to be oscillatory b) The voltage across the resistor at $t=0^+$ is zero c) The energy stored in the inductor in the steady state is zero d) The resistor current eventually falls to zero Ans: (b) Q23) A 1 μ F capacitor charged through a 2 kQ resistor by a 10V dc source. The initial growth of capacitor voltage will be at the rate c) 6.32 V/ms a) 3.16 V/ms b) $5.0 V/ms$ d) $10.0 V/ms$ $Ans(b)$ $Q24$) A series $R - C - L$ circuit is driven by an ac voltage source. Then the voltage across the following elements or the pair of elements cannot exceed the applied voltage a) C b) L $c) R$ $d)$ R and L $Ans(c)$ Q25) A series R-C circuit has a capacitor with an initial voltage of 11 V. A 15 V dc source is now connected across the R-C circuit. The initial rate of change of capacitor voltage can be a) 15 X 0.368 / RC b) 15X 0.632 / RC c) $11/RC$ $d)$ 4/ RC $Ans: (d)$ Q26) For the compensated attenuator of fig below, the impulse response under the condition $R_1C_1=R_2C_2$ is C_1 $V_i(t)$ $V_0(t)$

a) $R_2/(R_1+R_2)[1-e^{-tR_1C_1}]u(t)$ b) $R_2/(R_1+R_2)\delta(t)$ c) $R_2/(R_1+R_2)u(t)$ d) $R_2/(R_1+R_2)[1-e^{-tR_1C_1}] \delta(t)$ Ans: (b) Q27) What is v_c (o+)?

Q28) The switch K opened at $t = 0$ after the network has attained a steady state with the switch closed.
Find $vs(0+)$ across the switch ?

TO BE A TEACHER MEANS TOUCH HEART RATHER THAN HEAD

Q32) Given Initial charge in $C_0 = 500 \mu C$. In the steady state find charge in 1 μ f capacitor?

a) $2/3$ A $b)0$ c) $4/3$ $d)$ 1A Q35) What is $V_L(0+)$, when switch K is closed at t=0?

a) $2V$ $b) -2$ $c)0$ d) none Ans: (b) Q36) An impulse current 28 (t) A, with t in second, is made to flow through an initially relaxed 3F capacitor. The capacitor voltage at $T = 0 + i s$ a) $6V$ $b) 2V$ c) $2/3$ V d) zero Ans: (c)

Ans: (a)

a) $v(0+) = 1V$, $i(0+) = 0A$ d) $v(0+) = 1 V$, $i(0+) = 1 A$

 Q 41) In the circuit shown, the switch is moved from position A to B at time $t = 0$. The current i through the inductor satisfies the following conditions 1. i(0) = -8A 2. di / dt (t=0) = 3A / s 3. i(∞) = -4A the value of R $\overline{1}S$

a) 0.5 ohm b) 2.0 ohm c) 4.0 ohm d) 1 Ans: (a) ı $Q42$) In the circuit shown above, the switch is closed at $t = 0$. The current through the capacitor will decrease exponentially with a time constant

 $a) 0.5 s$ $b)$ 1 s c) $2s$ d) $10s$ Ans: (b) Q43) In the network shown, the switch is opened at $t = 0$. Prior to that, network was in the steady-state, Vs (t) at $t=0$ is

 $Ans(c)$

 $+V_S(t)$

c)
$$
2 \frac{d^2i}{dt^2} + 2 \frac{di}{dt} + i(t) = \cos t
$$
 $d) \frac{d^2i}{dt^2} + 2 \frac{di}{dt} + 2i(t) = \sin t$ Ans: (c)

Q48) For the circuit shown the switch is in position 1 for a long time and thrown to position 2 at t=0. At t=0⁺, the current i₁ is

- 2. The response of a network is decided by the location of
- a) Its zeros b) Its poles c) both zeros & poles d) neither zeros nor poles. Ans: (c)

3. The pole-zero configuration of a network function is shown. The magnitude of the transfer function will

11. The open-circuit transfer impedance Z_{21} of the two-port network is

a) $(Z_a - Z_b)/2$ b) $(Z_b - Z_a)/2$ c) $(Z_a + Z_b)/2$ d) $Z_a + Z_b$ Ans: (b) 12. Two networks are cascaded through an ideal buffer. If td₁ & td₂ are the delay times of two networks, then the over all delay time of the two networks together will be b) $\sqrt{(td_1^2 + td_2^2)}$ a) $\sqrt{\text{td}_1 \text{td}_2}$ c) $td_1 + td_2$ d) $(td_1 + td_2)/2$ Ans: (c) 13. The two-port network shown in fig. described by the relationships $V_1 = kV_2$ and $I_1 = kI_2$ its input impedance

 $a) R$ 14. A 2- port network is shown in fig. The parameter h_{21} for this network can be given by

 $a) - \frac{1}{2}$ $b) +1/2$ $d) + 3/2$ c) $-3/2$

Ans:(a)

Ans:(b)

15. For the circuit shown identify the correct statement , where Za is Z-parameters of top circuit, Zb is Z parameters of bottom circuit and Z is the Z parameters of complete circuit

a) for any value of R_1 and R_2 $Z = Z_a + Z_b$ b) If $R_1 = R_2 = 0$ then only $Z = Z_a + Z_b$ c) If R_1 and R_2 is equal to 1 ohm then only $Z = Z_a + Z_b$ d) None Ans: (b) 16. A two port network is reciprocal, if and only if a) $Z_{11} = Z_{22}$ b) BC - AD = -1 c) $Y_{12} = -Y_{21}$ d) $h_{12} = h_{21}$ Ans;(b)

17. The two – port network shown in the fig. is characterized by the impedance parameters Z_{11} , Z_{12} , Z_{21} and Z_{22} . For the equivalent Thevenin's source looking to the left of port 2, the V_T and Z_T will be respectively

a)
$$
V_T = \frac{Z_{11}}{Z_{11} + Z_g}
$$
 V_g ; $Z_T = Z_{22} - Z_{12}$
b) $V_T = \frac{Z_{12}}{Z_{11} + Z_g}$ V_g ; $Z_T = Z_{22} - Z_{12}$
c) $V_T = \frac{Z_{12}V_g}{Z_{11} + Z_g}$ $Z_T = Z_{22} + \frac{Z_{12}Z_{21}}{Z_{11} + Z_g}$
d) $V_T = \frac{Z_{12}V_g}{Z_{11} + Z_g}$; $Z_T = Z_{22} - \frac{Z_{12}Z_{21}}{Z_{11} + Z_g}$ Ans.(d)

18. In respect of the 2-port network shown in the fig. The admittance parameters are: $Y_{11} = 8$ mho, $Y_{12} = Y_{21} = -$ 6 mho and $Y_{22} = 6$ mho. The values of $Y_a Y_b$, Y_c (in units of mho) will be respectively

a) $2,6$ and -6 b) $2,6$ and 0 c) $2,0$ and 6 d) 2,6 and 8 $Ans:(c)$ 19. If the transmission parameters of the network are $A = C = 1$, $B = 2$ and $D = 3$, then the value of Z_m is

22. The network in the box shown displays the following z parameters: z_{11} = 50 ohms, z_{12} = -100 ohms, z_{21} = 500 ohms, and $z_{22} = 2.5$ k ohms. Determine the circuit required for z_L to insure maximum power transfer. Assume $f = 60$ Hz.

a)2.8k ohms. 26.5 µF in series. b)2.6k ohms, 26.5 µF in series c)2.8k ohms, 265 mH in series d)2.6 k ohms, 265 mH in series Ans: ©

23. Find Z₂₂ of the circuit shown in the fig: with dot sign at the top side of two windings

25. Find Y_{11} of the fig; shown

Ans: (c)

a)
$$
(R_1 + R_2)/2
$$
 b) $(R_2 - R_1)/2$ c) $(R_1 - R_2)/2$ d) $2 (R_1 - R_2)$ Ans:
\n34. The Y parameters of a four – terminal block are
\n
$$
\begin{array}{|c|c|c|}\n\hline\n4 & 2 \\
\hline\n1 & 1\n\end{array}
$$
\nAsingle element of 1 ohm is connected across

as shown in the given fig. The new Y parameters will be

NETWORK FUNCTIONS

- 1. The necessary and sufficient condition for a rational function of $T(s)$ to be driving point impedance of an RC network is that all poles and zeros should be
- a. Simple and lie on the negative axis in the s- plane b) Complex and lie in the left half of the s- plane
- b. Complex and lie in the right half of the s- plane d) Simple and lie on the +ve real axis of the s-plane

2. For an RC driving – point impedance function the poles and zeros a) Should alternate on real axis b) should alternate only on the real axis c) Should alternate on the imaginary axis d) can lie any where on the left half plane Ans: (b) 3. The transfer function of a passive circuit has its poles and zeros on a) Left and right halves respectively of the s-plane b) right and left halves respectively of the s-plane c) Right half of the s – plane d) left half of the s- plane. Ans: (a) 4. A realizable driving point function $N(s)$ can be expressed as follows: N (S) = KS / $(S^2+w_0^2)$ + F₁ (S) where F₁ (S) has no poles at S= \pm jw₀. The constant K a) may be complex b) must be real and positive c) must be real and negative d) must be real but may be positive or negative. Ans: $()$ 5. An LC one-port has two inductances and a capacitance connected in such a manner that the two inductances cannot be combined into one. The driving point impedance will have a) a zero at s=0 as well as at s= ∞ b) a pole at s=0 as well as at s = ∞ c) a zero at $s=0$ and a pole at $s=\infty$ d) a pole at $s = 0$ and a zero at $s = \infty$ Ans: $()$ 6. An RLC network has two poles which are complex conjugates and very close to the jw-axis. Its transient response a) is critically damped b) is over damped c) is under damped d) cannot be determined from this data Ans: () 7. An impedance function Z (s) is such that Re(Z (jw)) < 0 for $w_1 < w < w_2$ and Re(Z (jw)) > 0 for 0 \le w \lt w_1 , and $w_2 \le w \le \infty$. It a) can be realized by an RC network. b) can be realized by an RL network c) can be realized by an RLC network d) cannot be realized by an RLC network. 8. A gyrator has an admittance matrix $= 0$ G It synthesizes an inductor at its input terminals when $-G = 0$ terminated by a capacitor C. The magnitude of inductor is b) C/G^2 a) G^2C c) G^2/C $d)$ 2CG Ans: (b) 9. Match List --I with List --II and select the correct answer using the codes given below the Lists: List-I $List-II$ A. Internal impedance of an ideal current source is 1. Forced response of the circuit B. For attenuated natural oscillations, the poles of the 2. Natural response of the circuit Transfer function must lie on the E^2 C. A battery with an e. m. f. E and internal resistance 3. R delivers current to a load RL. Maximum power 4R transferred is E^2 D. The roots of the characteristic equation given 4

5. Left hand part of the complex frequency plan

 $2R$

Ans: (a)

Tutorial Problems

Known gaps

Group discussion topics.

References, Journals, websites and E-links

Text Books

1. Engineering circuit analysis by William Hayt and Jack E.Kemmerly, Mc Gra Hill Company, 6th edition

- 2. Circuits and Networks by A.Sudhakar and Shyammohan S Pilli, Tata Mc Graw-Hill
- 3.Electrical Circuits by A.Chakrabarthy, Dhanpat Rai & Sons

Reference Text Books

1.Network Analysis By M.E Van Valkenberg 2.Network analysis by N.Srinivasulu

Websites

.

1.www.umm.edu.ac.in

2.www.IEEE explore.com

Journals 1.IEEE spectrum 2.Electronics for you

Quality Control Sheets

To be attached

STUDENT LIST

Class / Section: EEE 2yr/1sem

Group-Wise students list for discussion topics

Closure Report:

- 1. Total Number of classes planned 60
	-
- 2. Total Number of classes actually taken 75
- 3. Total Number of students attended for the internal exam -
- 4. Total Number of students attended for the external exam -
- 5. Total number of students passed the exam -

Pass percentage

STUDENT LIST

Class / Section: EEE 2yr/1sem

